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Short communication

A simple parametric equation for pseudocoloring grey scale images keeping their original brightness progression

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Abstract

A new formalism to generate pseudocolor spectra is introduced. Therefore, a parametric path in the RGB-color cube is derived to guarantee constant brightness progression. The resulting spiral-like curve allows the generation of various color spectra by adjusting the spiral's frequency and phase. The formalism allows the determination of the resulting number of colors in dependency of both parameters. Therefore, these spectra are particularly useful to represent 12 bit data on common computer graphic adapters with 4096 displayable pseudocolor values instead of 256 displayable grey intensities. In this paper, the spectra are applied to medical X-ray images yielding an enhanced visualization of the diagnostic information.

Keywords: Visualization; Pseudocolor; Grey-to-color transformation

1. Introduction

Modern methods of medical diagnosis, such as X-ray imaging (including CT and DSA), ultrasound or MRI, contain information that is usually displayed as grey scale pictures. Diagnosis is often based on small luminosity differences, e.g. textures caused by tumours. There are two major reasons why those images may be visualized by pseudocoloring. The first reason is that, although the human eye is able to distinguish 10^{13} luminosity values, under usual illumination conditions, only less than 50 can be differentiated simultaneously [1]. However, the physiological properties of the eye permit much higher color sensitivity. Secondly, the original data in medicine often contains up to 4096 shades of grey (12-bit), while the true color graphic adapters (24-bit), which were usually used to display the images, quantize each direction of the RGB-cube with 8-bit resolution, yielding only 256 different grey values. Therefore, one can use pseudocoloring to emphasize the detailed structure in grey scale images.

A lot of pseudocolor mapping schemes are described in the literature [2]. Following the edges of the RGB-cube results in a constant-saturation pseudocoloring. Clark and Leonhard [3] translated the grey scale into a rainbow-like color spectrum with respect to a physiological homogeneous brightness (Fig. 1). A major drawback of those mapping schemes is their piecewise definition, which complicates calculation and implementation. Concerning medical X-ray images, constant-brightness as well as constant-saturation pseudocoloring extinguishes the relationship between high and low X-ray intensities, and thus leads to an essential loss of diagnostic values [4].

In this paper, a parametric grey-to-color transformation is introduced that, on the one hand, maintains the original progression of brightness and, on the other, generates plenty of color changes to make use of the eye's capability to distinguish many colors at the same time. In the next section, the parametric equation for pseudocoloring in the continuous RGB-domain is derived. The length of the parametric curve can easily be calculated, and is used in Section 3 to approximate the maximal number of different pseudocolors in the discrete RGB-domain. The pseudocoloring of medical X-ray images is exemplified in Section 4 before the results are discussed in Section 5.

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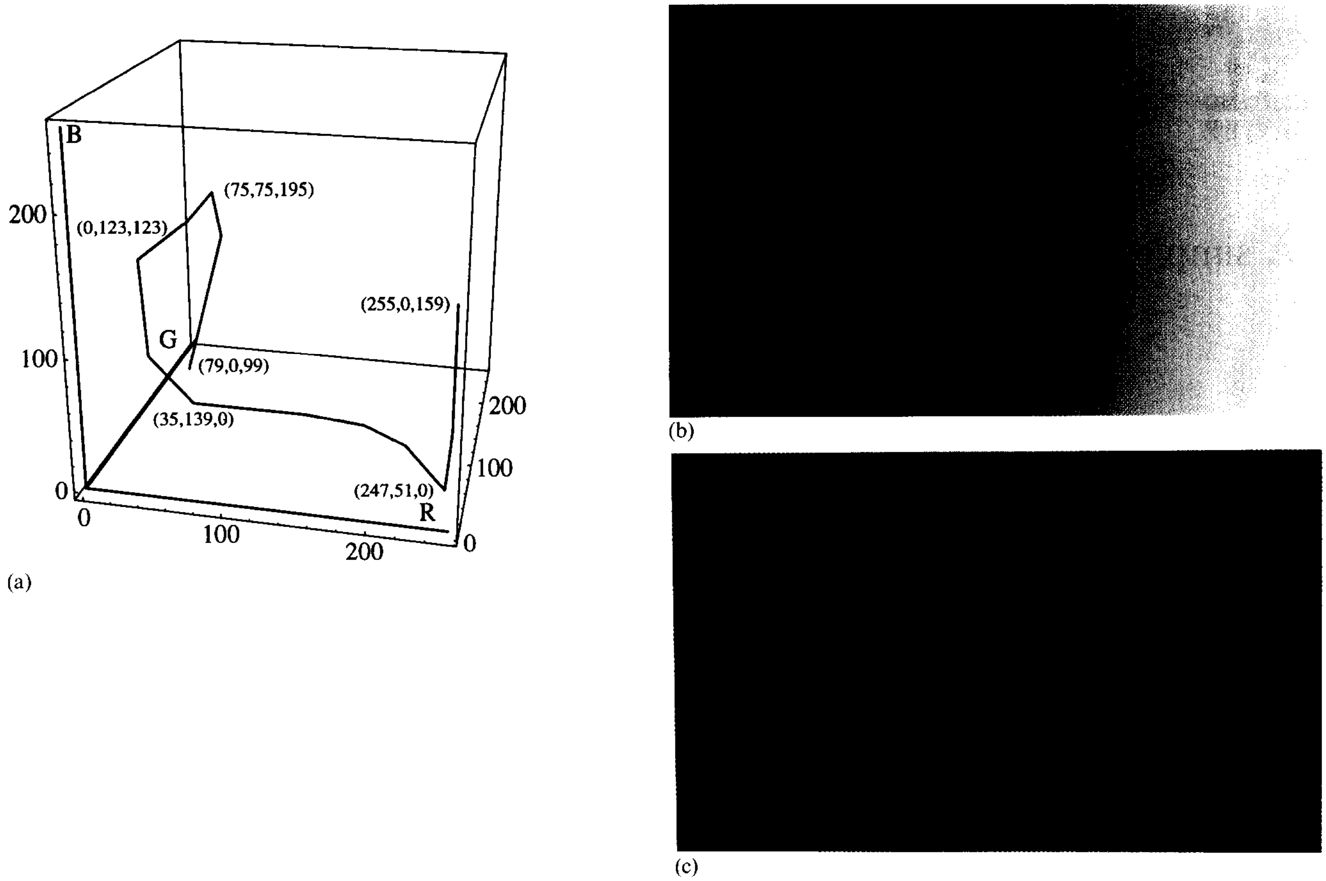

Fig. 1. The transformation of the grey scale ramp (b) to a rainbow-like color spectrum (c) as proposed by Clark and Leonhard [3] is shown in the RGB-cube (a). The piecewise linear curve was found experimentally with respect to a uniform progression of hue and constant brightness.

2. The spiral's formalism

Pseudocoloring can be described mathematically by a transformation curve in a color space (Figs. 1(a), 2(a)). The curve is equidistantly sampled to create as many points as there are input grey values. Each grey value is mapped to the specific color defined by the coordinates of the corresponding sample point in the color space.

Using the RGB-model, those mapping schemes can directly be used as look-up tables (LUT) for the computer graphics adapter. The main diagonal in the RGB-cube from black to white represents ascending grey values (intensities). To realize both color and continuous-brightness, the transformation curve should follow a spiral-like path along this diagonal.

In a continuous three-dimensional domain a spiral-like curve along the z-axis with the parameter t is given by

$$\vec{v}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} r(t)\sin(\omega t + \varphi) \\ r(t)\cos(\omega t + \varphi) \\ z(t) \end{pmatrix}. \tag{1}$$

The phase φ defines the direction, and frequency ω is related to the number of rotations of the spiral around the z-axis. The function $r(t)$ determines the form of the spiral, and $z(t)$ its progression. To use Eq. (1) for pseudocoloring, the z-axis has to be mapped to the main diagonal of the RGB-cube. This can be realized by the matrix $\mathbf{M}$:

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \mathbf{M} \cdot \vec{v}, \tag{2}$$

rotating the coodinate system $(x, y, z)^{\mathrm{T}}$ into $(R, G, B)^{\mathrm{T}}$.

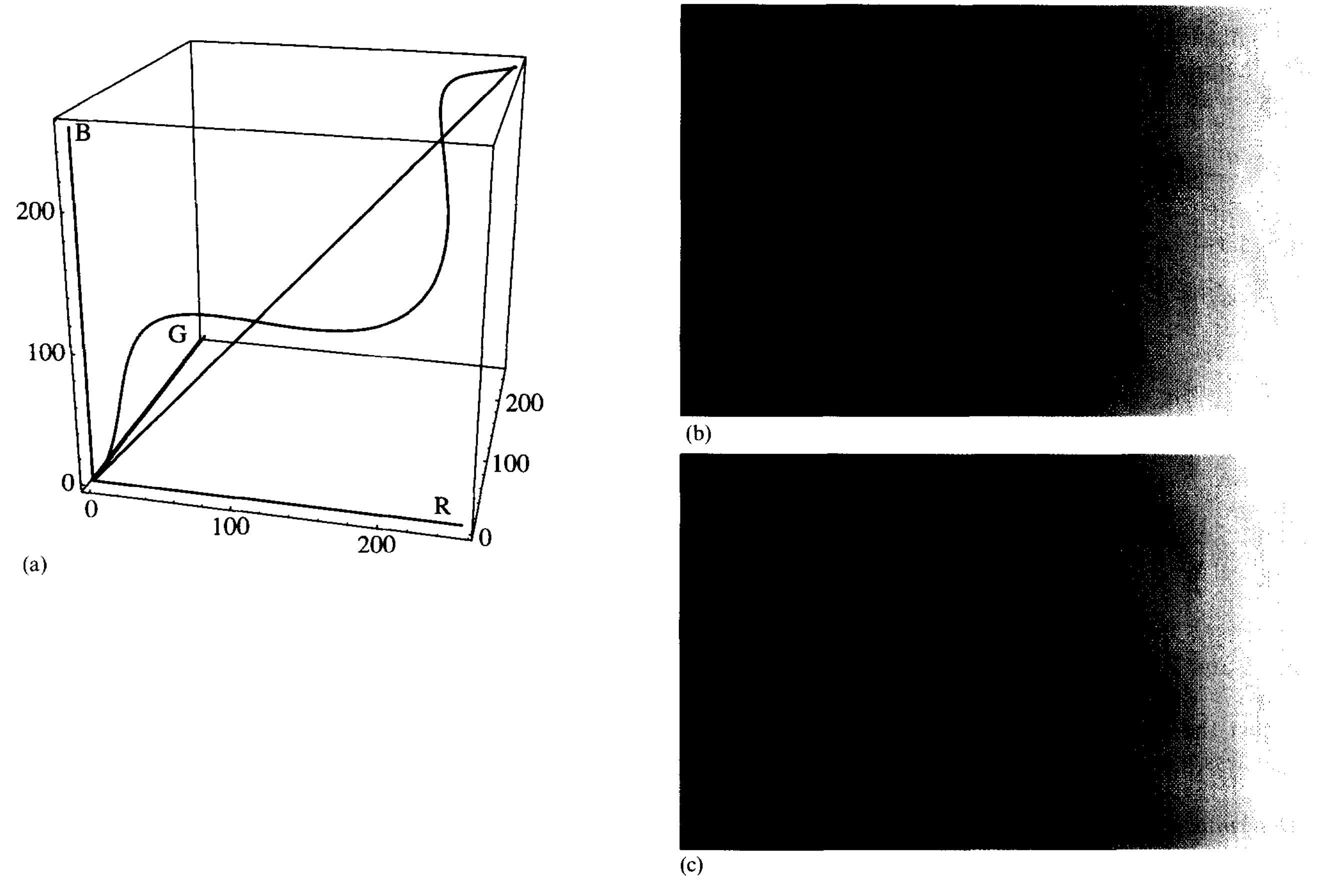

Fig. 2. Using Eq. (7) with the functions $r(t) = r_2(t)$ and $z(t) = z_1(t)$ and the parameters $\omega = 2\pi \cdot 1.3$ and $\varphi = 3.0$, the spiral curve as shown in (a) is obtained. Part (c) shows the corresponding pseudocoloring of the grey scale ramp (b).

2.1. Determination of matrix *M*

Any rotation of a cartesian coordinate system is given by:

$$\mathbf{M} = \begin{pmatrix} \cos\beta + a_x^2(1-\cos\beta) & a_z\sin\beta + a_x a_y(1-\cos\beta) & -a_y\sin\beta + a_x a_z(1-\cos\beta) \\ -a_z\sin\beta + a_y a_x(1-\cos\beta) & \cos\beta + a_y^2(1-\cos\beta) & a_x\sin\beta + a_y a_z(1-\cos\beta) \\ a_y\sin\beta + a_z a_x(1-\cos\beta) & -a_x\sin\beta + a_z a_y(1-\cos\beta) & \cos\beta + a_z^2(1-\cos\beta) \end{pmatrix} \quad (3)$$

where β determines the rotation angle and a_x, a_y and a_z the cosine of the solid angles between the rotation axis g and the coordinate axis x, y and z, respectively. Referring to Fig. 3(a), one can easily determine the cosine of the solid angles, because g must stay in the (x, y)-plane:

$$a_x = \cos 45^\circ = \frac{1}{\sqrt{2}}, \quad a_y = \cos 135^\circ = -\frac{1}{\sqrt{2}}, \quad \text{and} \quad a_z = \cos 90^\circ = 0. \quad (4)$$

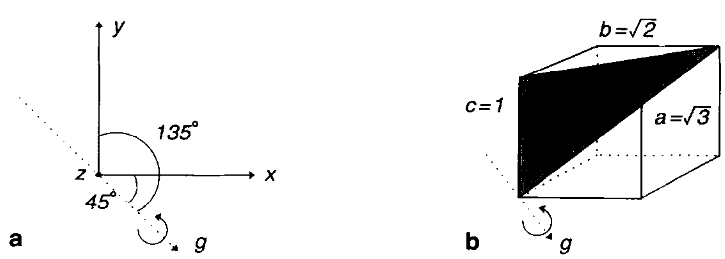

Fig. 3. An appropriate rotation of the coordinate system is achieved if the rotation axis g stays in the (x, y)-plane (a). Therefore, the solid angles between x, y and z and g are 45°, 135° and 90°, respectively. The rotation angle β can be determined considering the shaded triangle (b).

Fig. 3(b) illustrates the rotation angle β. The length of the edges of the RGB-cube is set to 1, resulting in the length of the planar diagonals being $\sqrt{2}$ and the length of the main diagonal being $\sqrt{3}$. The angle β of the skew-angled triangle with the edges $a = \sqrt{3}$, $b = \sqrt{2}$ and $c = 1$ is given by the cosine theorem

$$\cos(\beta) = \frac{a^2 + c^2 - b^2}{2ac} = \frac{1}{\sqrt{3}} \Rightarrow \sin(\beta) = \frac{\sqrt{2}}{\sqrt{3}}. \tag{5}$$

With Eqs. (4) and (5), the rotation matrix **M** can be determined:

$$\mathbf{M} = \begin{pmatrix} \cos\beta + a_x^2(1-\cos\beta) & a_x a_y(1-\cos\beta) & -a_y \sin\beta \\ a_y a_x(1-\cos\beta) & \cos\beta + a_y^2(1-\cos\beta) & a_x \sin\beta \\ a_y \sin\beta & -a_x \sin\beta & \cos\beta \end{pmatrix} = \frac{1}{2\cdot\sqrt{3}} \begin{pmatrix} 1+\sqrt{3} & 1-\sqrt{3} & 2 \\ 1-\sqrt{3} & 1+\sqrt{3} & 2 \\ -2 & -2 & 2 \end{pmatrix} \tag{6}$$

and the parametric Eq. (2) for pseudocoloring finally results in

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \frac{1}{2\cdot\sqrt{3}} \begin{pmatrix} 1+\sqrt{3} & 1-\sqrt{3} & 2 \\ 1-\sqrt{3} & 1+\sqrt{3} & 2 \\ -2 & -2 & 2 \end{pmatrix} \cdot \begin{pmatrix} r(t)\sin(\omega t + \varphi) \\ r(t)\cos(\omega t + \varphi) \\ z(t) \end{pmatrix}, \tag{7}$$

with $0 \le t \le 1$. Note that, for the sake of mathematical simplicity, the color-axes in Eq. (7) are normalized to 1, whereas in Figs. 1 and 2 the RGB-cube is scaled to 8-bits.

In Eq. (7), the phase φ determines the starting color, and frequency ω the dynamic of color changes. In addition, one can adjust the degree of coloring by the orthogonal distance $r(t)$ between the spiral curve and the main diagonal. Nevertheless, the functions $t(t)$ and $z(t)$ must ensure the spiral to reside within the finite RGB-cube.

2.2. *Determination of functions* r *and* z

The largest distance $r(t)$ is limited by the remaining volume within a rotating RGB-cube, if the main diagonal is considered as the axis of rotation. Fig. 4(a) shows the RGB-space. Due to the symmetry of the cube, the remaining volume is determined by two planar triangles, e.g. *A-B-C* and *A-D-B*. The one-dimensional projection of the rotating cube is shown in part (b) of this figure. The triangular planes overlay to a M-formed shape. In this projection, the distance $r(t)$ must stay within both triangles. Therefore, the largest slope $dr/dt|_{t=0}$ is equivalent to $\tan(\gamma)$ of the skew-angled triangle *A-B-C*. Again, the slope $\tan(\gamma)$ is given by the cosine theorem

$$\cos(\gamma) = \frac{a^2 + b^2 - c^2}{2ab} = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \tan(\gamma) = \frac{1}{\sqrt{2}}. \tag{8}$$

Taking into acount the $\sqrt{3}$-stretch of the t-axis, the piecewise linear function $r_1(t)$ and the continuous function $r_2(t)$ in Fig. 4(b) are given as

$$r_1(t) = \sqrt{\frac{3}{2}} \cdot \begin{cases} t & \text{if } 0 \le t \le 1/2 \\ 1-t & \text{elsewhere} \end{cases} \quad \text{and} \quad r_2(t) = \sqrt{\frac{3}{2}} \cdot t(1-t). \tag{9}$$

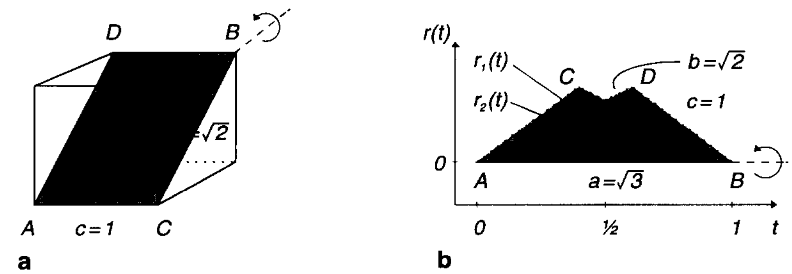

Fig. 4. The largest distance $r(t)$ between the spiral and the main diagonal *A-B* equals the remaining volume within the rotating RGB-cube. In the one-dimensional projection (b) of the rotating cube (a), the intersection triangles like *A-B-C* and *A-D-B* overlap to an M-formed shape. Both the piecewise linear and the continuous distance function $r_1(t)$ and $r_2(t)$, respectively, remain within the triangles (see Eq. (9)).

Finally, $z(t)$ must be chosen so that $z(t)|_{t=0} = 0$ and $z(t)|_{t=1} = \sqrt{3}$. In the easiest case, $z(t) = z_1(t)$ is linear:

$$z_1(t) = \sqrt{3}t, \tag{10}$$

but $z(t)$ can also be used to gamma-correct the intensity scale.

Using r_i and z_1 (see Eq. (9) and (10), respectively), the only parameters remaining in Eq. (7) are ω and φ. The variation of both parameters results in many psuedocolor spectra (Fig. 2). For a given (ω, φ)-tuple, the grey-to-color LUT can easily be generated by scaling t to the number of input grey levels, and calculating as many equidistant spiral points as is required for output color values.

3. The spiral's length

In practice, X-ray films are often digitized with a 12-bit quantization, but reduced to 8-bit (256 shades of grey) for display. To avoid this loss of valuable information, the spiral's formalism can be used to generate an appropriate transformation into 4096 displayable pseudocolors. The required length of the spiral can be realized using high frequencies ω.

The simplicity of the parametric formalism has the beneficial effect that the length of the spiral can be determined *a priori*. In the continuous space, the length of a parametric curve $\vec{v}(t) = (x(t), y(t), z(t))^{\mathrm{T}}$ between the points t_1 and t_2 is given as

$$l = \int_{t_0}^{t_1} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt. \tag{11}$$

Using Eq. (1) and the identity $\cos^2(\cdot) + \sin^2(\cdot) = 1$, Eq. (11) results in:

$$l(\omega) = \int_0^1 \sqrt{\left(\frac{dr}{dt}\right)^2 + (r(t)\cdot\omega)^2 + \left(\frac{dz}{dt}\right)^2}\, dt. \tag{12}$$

With $r_2(t)$ and $z_1(t)$ defined in Eq. (9) and (10), respectively, Eq. (12) yields:

$$l(\omega) = \sqrt{\frac{3}{2}} \int_0^1 \sqrt{(1-2t)^2 + t^2(1-t)^2\omega^2 + 2}\, dt \quad \text{with} \quad \lim_{\omega\to\infty} l(\omega) = \omega \frac{1}{2\sqrt{6}}. \tag{13}$$

Let us now consider the discrete RGB-space with N^3 voxels and, thus, N quantizing steps in each direction: red, green or blue. To guarantee that the voxels of the RGB-cube are crossed by the spiral only once, the number of rotations

$$n = \frac{\omega t}{2\pi}\bigg|_{t=1} \tag{14}$$

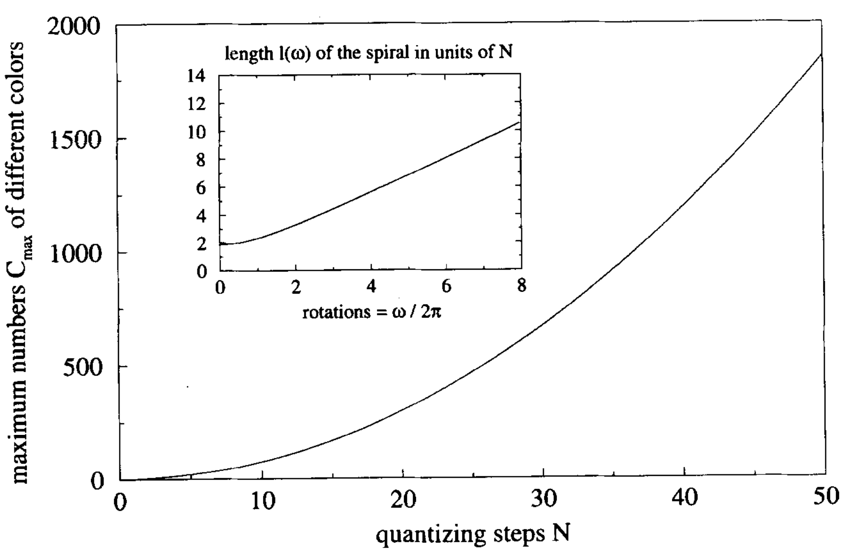

Fig. 5. The maximum number of different colors $C_{\max}$ has a parabolic dependence on the quantizing steps N of each direction of the RGB-cube (see Eq. (17)). The inset shows the length $l(\omega)$ of the spiral curve as a function of rotations (see Eq. (13)).

must be limited. To approximate the cut-off frequency $\omega_{\max}$, a single turn of the spiral is considered. After one turn, equivalent to the increase $\Delta t = 2\pi/\omega$, at least the maximum distance

$$d = \frac{\sqrt{3}}{N} \tag{15}$$

between two neighboring voxels must be covered along the main diagonal, thus $\Delta z(t) > d$. Using z_1 (see Eq. (10)), one gets with $\Delta z(t) \approx \partial z(t)/\partial t \, \Delta t$ the cut-off frequency

$$\omega_{\max} = 2\pi N, \tag{16}$$

and therefore, one can realize at least:

$$C_{\max} = \frac{l(\omega_{\max})}{d} = \frac{2\pi N}{2\sqrt{6}} \cdot \frac{N}{\sqrt{3}} \approx 0.74N^2 \tag{17}$$

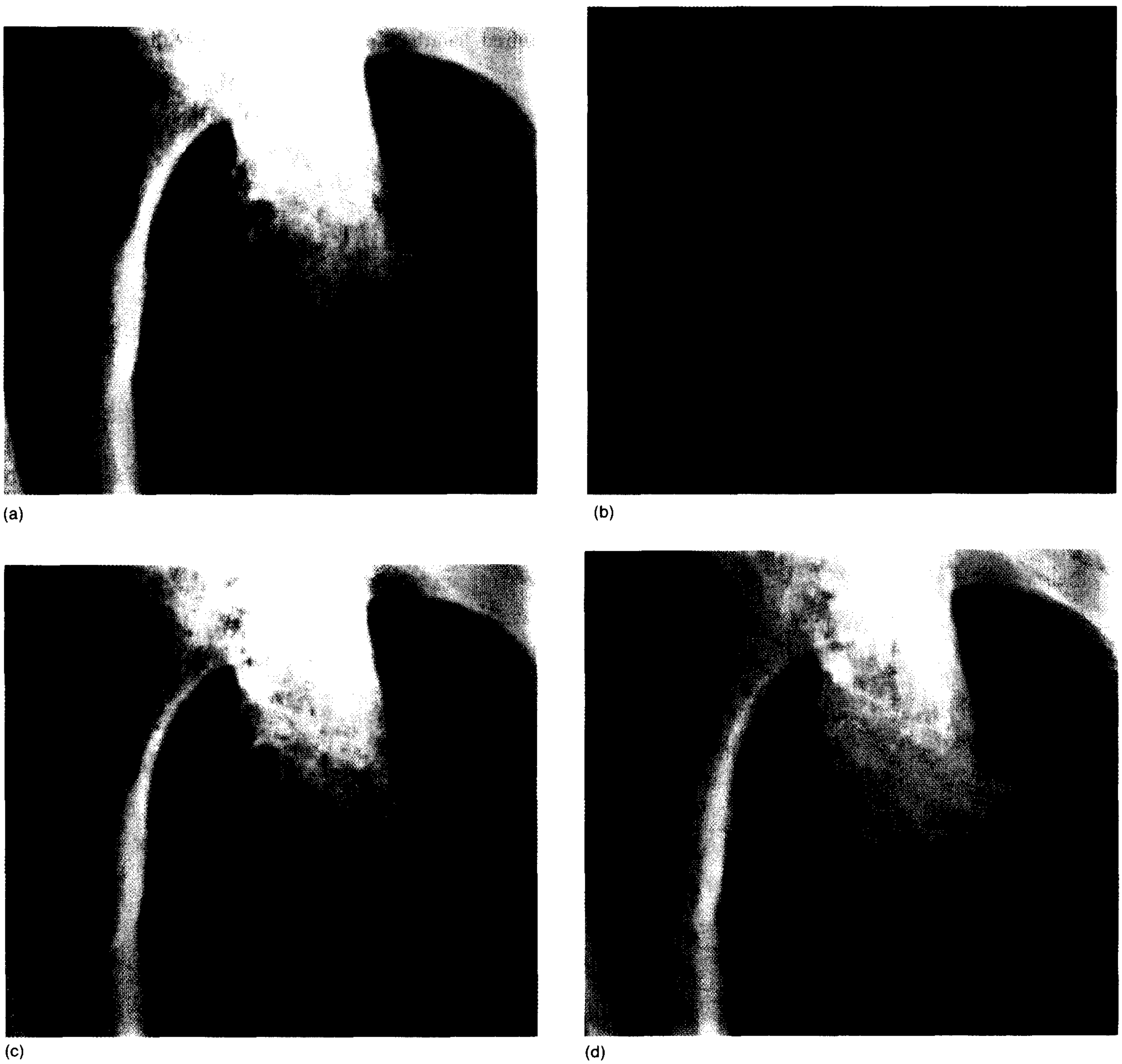

Fig. 6. (a) Original X-ray image of a pelvic bone metastasis after radiotherapy; (b) pseudocolored image of (a) using the rainbow method [3]; (c) (d) results of pseudocoloring using the spiral method with the parameters $\omega_{(C)} = 2\pi \cdot 1.3$, $\varphi_{(C)} = 3.0$ and $\omega_{(D)} = 2\pi \cdot 3.0$, $\varphi_{(D)} = 0$. The structure of the sclerose is most conspicuous in part (d).

different colors (Fig. 5). Furthermore, the required number of rotations n to realize $C = l/d$ colors is given by Eqs. (14), (13) and (15) as

$$n = \frac{C}{N} \cdot \frac{\sqrt{6}\sqrt{3}}{\pi} \approx 1.35 \cdot \frac{C}{N} \quad \text{with} \quad n_{\max} = 1.35 \cdot \frac{C_{\max}}{N} \approx N. \tag{18}$$

Eqs. (17) and (18) may be used to assess the capacity of the spiral's formalism for pseudocoloring. On the one hand, with $N = 256$, for instance, one can depict about 50,000 ($\approx$16-bit) different brightness values. On the other hand, one needs only 22 rotations of the 256 possible ones to represent a 12-bit image with $C = 4096$ pseudocolors.

4. Application to medical X-rays

Fig. 6 shows examples of the enhancement of a scelerosed structure in the pelvic bone using spiral pseudocoloring. Although the lightness axis is approximated by the black to white diagonal of the RGB-space, the detailed image structure and the brightness progression is well pronounced. The usefulness of this algorithm applied to dental radiographs is discussed elsewhere [4].

5. Conclusion and discussion

The spiral-like path in the RGB-cube maps grey scale images to pseudocolor images keeping their original brightness progression. This feature is essential in medical imaging. The parametric formalism presented in this paper is easily computed in the RGB-space and allows the *a priori* calculation of the number of resulting colors, depending on the spiral's frequency ω. Adapting ω and the phase φ, various pseudocolor spectra can be generated.

Better results could be obtained if other color models were used. Within the HLS-system [5], the lightness is directly represented by one of the axes. Because the HLS-space is of a high geometric symmetry, Eq. (7) could easily be transferred by dropping the matrix **M**. To visualize images which have been pseudocolored in the HLS-space, an additional transformation back to the RGB-space has to be performed.

Further improvement may be achieved by taking into account the physiological color sensitivity of the human eye. Color spaces like *Lab* or *Luv* model human perception accurately, but completely lose the mathematical simplicity of the proposed pseudocoloring. The AR_gY_b (Achromatic, Red-green, Yellow-blue) color system proposed by Naiman [6] seems to be a promising compromise. Therefore, the transfer of the spiral's formalism to the AR_gY_b system for pseudocoloring medical X-ray images is the aim of future research.

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