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REVIEW

Computer-based registration for digital subtraction in dental radiology

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Objectives: (1) To review computerized *a posteriori* techniques for geometry and contrast registration prior to digital subtraction in dental radiography; (2) to define a uniform notation for their methodological and technical classification and based on this key code; (3) to derive criteria for successful application of computer-based *a posteriori* registration for routine clinical subtraction.
Methods: All techniques are classified with respect to the (1) dimension of geometry registration; (2) origin; (3) abstraction level, and (4) linkage of features used for registration of geometry; (5) elasticity; (6) domain, and (7) parameter determination of the geometrical transform used; (8) interaction of geometrical registration; as well as (9) origin of features, (10) model of transform, and (11) interaction of procedure for contrast correction.
Results: With respect to clinical practicability, superior registration techniques are based on the low level abstraction of intrinsic features for both geometry and contrast registration. By approximately linking the features, a global projective transform should be generated for geometry registration by automatic methods, while automatic contrast correction should be non-parametric. This challenge is met only by one out of 36 published algorithms. Hence, although numerous computer-based techniques have been published, only a few of them are applied more than once in practice.
Conclusion: The key code proposed in this paper is useful for technical classification of *a posteriori* registration methods in dental radiography and allows their objective comparison. Further investigations will focus on standardization of practicable procedures to evaluate the robustness of competing methods.

Keywords: radiographic image enhancement; subtraction technique; digital radiography, dental; radiography, dental

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1 Introduction

The detection of small changes in serial radiographs has been achieved using subtraction methods since their introduction by Zeidses des Plantes in the 1930s. This so-called 1st generation of subtraction systems employs photographic techniques for the subtraction of *a priori* registered radiographic films that are aligned manually (Figure 1, left).

The 2nd generation performs digital subtraction by means of computer. One of the earliest methods in dental radiology was reported by Gröndahl *et al.* in the early 1980s (below referred to as Gröndahl's method[122]). Based on standardized radiographs, the reference (baseline) radiograph is digitized, converted to an exact positive image by the computer and displayed on a television (TV) screen. A TV-camera is then connected to the same screen and the subsequent (follow-up) image in its negative modality is superimposed on the positive reference image. By means of a device permitting rotation and translation of the subsequent radiograph, it is aligned and then digitized.[122] Ortman *et al.* add a second stage to this manual adjustment procedure where both images are presented sequentially in a 'flicker'-mode.[48] Brägger and coworkers go further and suggest the use of three stages to register the images during digitization. Coarse registration is done by a 'chessboard'-mode where 50% of each radiograph is displayed on 50% of the monitor field,[110] while in earlier work, automatically outlined edges for manual superimposition during the TV-camera-based digitization are used.[31]

The architecture of 2nd generation systems is shown schematically in Figure 1 (middle). Similar to the 1st generation, all 2nd generation systems are based on mechanical stabilization of imaging geometry, principally, the relative position of the subject to the source and receptor. Film alignment is performed manually during the digitization process to ensure the discrete pixel grid in both radiographs correspond. If quantitative interpretation is performed, the digital images are corrected for contrast differences before subtraction.[52,53] Numerous studies, starting in the early 1980s, have proven that digital subtraction radiography is capable of exquisite sensitivity to small changes, provided that the experimental conditions can be held constant.[2,8,19]

The 3rd generation of subtraction systems based on direct digital radiography,[2,9,25,28] with either CCD or storage phosphor receptors, was introduced in the late 1980s. Even though imaging geometry still is controlled mechanically, an alignment procedure, at least for compensation of the planar rigid transforms (i.e. translations and rotations), is required *after* the digitization or digital acquisition of radiographs

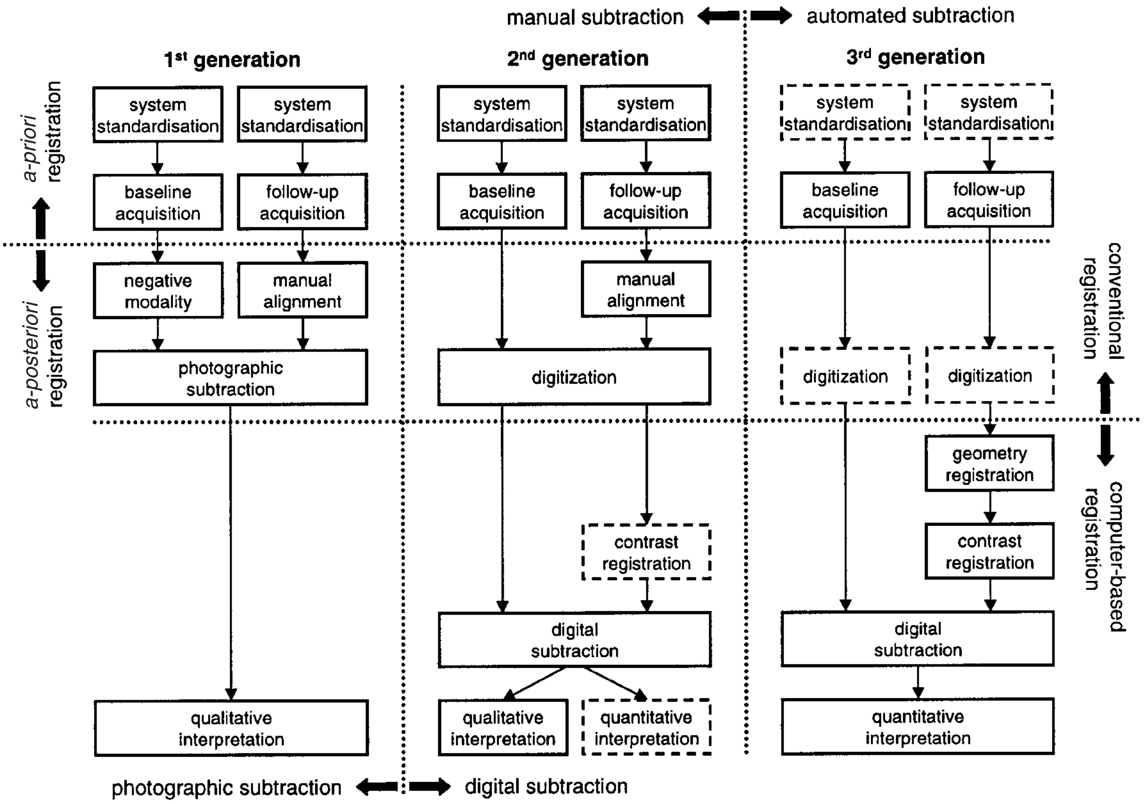

Figure 1 Classification of subtraction methods. Dashed boxes indicate optional algorithms

(Figure 1, right). Hence, all 3rd generation systems perform the *a posteriori* registration by means of computer software. Furthermore, they are not restricted to rigid geometric transforms. Novel approaches to 3rd generation applications are based on more sophisticated models, such as affine or projective geometry, in order to compensate for less standardized imaging geometry by more computational complexity. In other words, *a priori* registration (mechanical standardization) is replaced by *a posteriori* registration (retrospective image processing). To date, several new methods have been introduced[22,23,27] and methodological papers on computer-based registration appear continuously in the literature.[32,45,51,62,64] However, as far as clinical diagnosis and treatment is concerned, the situation is unchanged. There are two reasons for this. On the one hand, *a priori* registration is expensive and too cumbersome for clinical use; on the other, selection of the proper technique is difficult because they lack comparability. So far, the results of computerized *a posteriori* registration have been demonstrated in numerous situations using *in vivo*, *in vitro* or phantom images in combination with a range of artificial manipulations. Hence, systems for 3rd generation subtraction are neither comparable nor routinely available. Therefore, about 20 years after the initial development of digital subtraction in dental radiology, the technique is still seldom performed in clinical practice.[5]

This review addresses the lack of comparability of computer-based *a posteriori* registration methods by introducing a uniform terminology (Section 2) and a technical classification of 3rd generation systems (Section 3). Based on this classification, the requirements for clinical applications can be derived. Published procedures are reviewed and classified (Section 4) before methods to compare the clinical power of registration techniques and the results are discussed in Section 5. The references are grouped into papers of general interest (Section 6.1), definitions (Section 6.2) and applications (Section 6.3) of 3rd generation registration systems, and other papers of interest (Section 6.4). Finally, a glossary provides brief definitions of technical terms used in this paper.

2 Terminology

According to Samarabandu *et al*., it is necessary to differentiate between *a priori* registration techniques that are used to stabilize projection geometry *before* image acquisition and *a posteriori* registration techniques required for subtraction of non-standardized (free-hand) images *after* their acquisition.[21] However, *a posteriori* registration is also applied to images obtained by imaging techniques with *a priori* registration to improve the match (Figure 1). Furthermore, conventional registration is done by hardware on X-ray films while computer-based registration is done by software after digitization.

2.1 *A priori registration*

Early subtraction techniques were based only on *a priori* registration to guarantee appropriate alignment of the radiographs.[10] Mechanical coupling of tube and film was achieved by individual adjustment aids, such as bite blocks,[111,117,136] cephalostats,[127] or combined filmholder and X-ray beam manipulators.[143] Araki *et al.* described a standardized lateral oblique projection of the mandible.[106] Such systems for *a priori* registration have been perfected by means of computer-assisted real-time measurement of tooth position and automated tube positioning by a robotic arm[71,112] or electronically guided alignment systems based on force-sensitive sensors.[124] However, *a priori* registration is expensive, cumbersome in clinical use, and limited in precision.[136] While the alignment error for bitewing radiographs using uncoupled positioning devices or individual bite blocks has been shown to be less than 2°,[52,105,117] 2.5°,[137] or 3°[135] in clinical circumstances, small angular discrepancies of only 0.7°[125] or 1°[115,135] on interpretation of subtraction radiographs has a significant effect. Based on *in vitro* analysis of alveolar bone changes, Shrout *et al.* have reported, in a more recent investigation, that alignment variations up to 5° may be acceptable in clinical studies if dental films are digitized with 50 μm spatial resolution.[138] Nevertheless, *a priori* registration may not guarantee sufficient alignment precision for subtraction.

2.2 *A posteriori registration*

During the last decade, *a posteriori* (retrospective[17]) registration techniques have become increasingly important. A global survey of such techniques has been undertaken by Brown.[4] Van den Elsen *et al.*[6] and, more recently, Maintz and Viergever[16] have reviewed methods of registration in medical imaging. *A posteriori* motion correction in digital subtraction angiography, which is closely related to digital subtraction imaging in dental radiology, was reviewed recently by Meijering *et al.*[17] Several criteria have been suggested in these reviews for the classification of *a posteriori* techniques. Based on these ideas, Section 3 presents an unambiguous analysis, uniform notation and complete coding for the classification of both geometry and contrast registration techniques in dental radiology.

2.3 *Computer-based registration*

In computer-based registration, both the geometry and contrast of individually standardized or free-hand radiographs are adjusted by means of computer software (Figure 1, lower right). Mathematically, a digital grayscale image f with dimension $(X \times Y)$ is defined as a function

$$f : \underline{X} \times \underline{Y} \rightarrow \underline{G} \qquad (1)$$

mapping each pixel (x,y) in a defined range, where $0 \leqslant x < X$, $x \in \underline{X} \subset \mathbb{R}$, and $0 \leqslant y < Y$, $y \in \underline{Y} \subset \mathbb{R}$, onto a specific gray-value $g \in \underline{G} \subset \mathbb{N}$, $0 \leqslant g < G$ out of the value range $\underline{G}$. From Equation 1, *a posteriori* image registration is defined as a transform

$$T : f \rightarrow f' \tag{2}$$

of the initial image into the corrected image, $f(x,y)$ and $f'(x',y')$, respectively.

2.3.1 Geometric registration In particular, geometric registration maps the coordinate system of the baseline image onto that of the follow-up radiograph or vice versa. In two dimensions, the point (x,y) is transformed into the point (x',y'). This can be written by two transforms T_x and T_y

$$\begin{aligned} x' &= T_x(x,y) \\ y' &= T_y(x,y) \end{aligned} \tag{3}$$

Therefore, geometric registration addresses the range of definition of the discrete image and proper interpolation techniques are required for its implementation.[14,140]

2.3.2 Contrast registration The contrast registration T_c operates on the range in value $\underline{G}$ of the image. For that, $\underline{G}$ is taken as the definition range of the transform T_c that maps the original image values g onto the contrast-registered values. Usually, T_c is performed subsequent to geometric registration and hence, from Equations 2 and 3 we obtain

$$f'(x',y') = T_c(f(x',y')) = T_c(f(T_x(x,y), T_y(x,y))) \tag{4}$$

3 Technical classification of methods for *a posteriori* registration

3.1 *A posteriori registration of geometry*

On the basis of previous work, we can define four major criteria for the differentiation of *a posteriori* geometric registration techniques: dimension, feature, transform and interaction.[13] These criteria are hierarchically subdivided into eight categories:

- dimension
- origin of features
- level of abstraction of features
- linkage of features
- elasticity of transform
- domain of transform
- determination of transform parameter
- interaction

3.1.1 Dimension Registration techniques differ in the dimension of their definition range. One-dimensional (1D) registration problems often occur in comparison of 1D biological functions, such as evoked potentials. A 1D registration method also can perform a temporal match on a series of spatially consistent images, e.g. in ultrasound imaging. However, registration of two images $f_1(x,y)$ and $f_2(x,y)$ is in general two-dimensional (2D). This also holds for image sequences $f_t(x,y)$ or sliced volume data $f_z(x,y)$ if the acquisition time t or the slice number z is processed sequentially. Note that almost all methods classified in Section 6.2 except[30,31,37,46,48,52,53,58,59,61] are 2D. The registration problem becomes three-dimensional (3D) if several or all slices are processed simultaneously.[37,58] In this case, 3D image data, is denoted by $f(x,y,t)$ or $f(x,y,z)$ for time series or volume data, respectively. Four-dimensional (4D) registration problems, which arise from matching of volume sequences $f(x,y,z,t)$, as well as other high-order techniques are seldom seen in dental radiology. Hybrid methods, such as multi-modal (inter-modal[29]) and multi-dimensional registration of 3D surfaces with 2D projections,[123] have not yet been reported in dentistry applications.

3.1.2 Features A feature is a well defined property that is occasionally obtained from the entire image but more often from parts of the image or pixels. *A posteriori* registration techniques are based on certain features. For the feature criterion, we distinguish the categories origin, level of abstraction, and linkage.

Origin The origin of features is either intrinsic or extrinsic. 'Intrinsic' features are patient-related properties, such as teeth and implants, the cemento-enamel junction (CEJ), or simply the bony structure, that show local characteristics in the X-ray image. 'Extrinsic' features are induced by artificial objects that are added to the patient prior to image acquisition, such as metal grids[60] or wire frames.[62] However, digital subtraction in dental radiology mostly is based on intrinsic features.

Note that the intrinsic *vs* extrinsic classification differs from the internal (intra-corporal) *vs* external (extra-corporal) nomenclature. This point is also discussed by van den Elsen *et al.* to distinguish whether a feature point is placed inside or outside the patient's body.[6] Extrinsic features do not necessarily need to be external. For example, contrast agents for angiography or MRI are extrinsic but internal features. More confusing, Brown has swapped the definition of intrinsic and extrinsic origin of features.[4]

Level of abstraction Image features also are classified by their level of abstraction (Figure 2). Image pixels are the smallest and therefore the most detailed unit for describing an image. The 'raw data' approach involves the entire image,[42,51,57] while 'pixel'-based approaches select only a few pixels with which to operate.[32,38,43–45,47,49,50,55,60,63] Each pixel is defined by its coordinate and brightness. At least two pixels are required to define local 'contrast'. Edges or lines that rely on the contrast level of abstraction are used also for computer-based *a posteriori* registration.[41,56,58] Although edges or lines usually are defined by more than two pixels, they are 1D structures. Further increase of dimensionality and hence of the number of pixels under consideration, allows the definition of 'texture' as a certain 2D pseudo-periodicity of local contrast. Cancellous bone exemplifies the texture level of abstraction in intraoral radiography. Textures have an area but not necessarily a well-defined shape. Although texture-based registration is not reported in dental radiology, applications can be found in mammography[141] and microscopy.[134] Multiple textures are concatenated to define image 'regions' or segments.[54] Note that the direct neighborhood of all pixels linked together to a certain segment is the only requirement at this stage of abstraction. Fisher and coworkers defined eight types of surface for region-based features which were applied to *a posteriori* registration of range data.[37] Employing *a priori* knowledge, regions or segments can be identified as 'objects'. Teeth or implants[130] are large-sized objects while the CEJ[33–36,39] or root apices[33,34,39,63] are small-sized. Finally, spatial or temporal relationships between objects define 'scenes'. In a non-dental application, Fritsch and coworkers define cores from image objects that provide means for describing object position and object-subfigure relationship at the scene level of abstraction.[118]

Note that the transition from the raw data level of image bitmaps to the scene level of image symbols dramatically reduces concrete information, such as image details. This increasing degree of abstraction equals, for example, the description of the dental status from a panoramic radiograph. While extrinsic features mostly are found on the object level, e.g. skin markers and metal grids or frames, intrinsic features cover the entire range from raw data up to the scene level of abstraction for icon up to symbolic image description, respectively (Figure 2).

The definition of levels of features of increasing complexity or abstraction is not consistent in the literature. Mol and van der Stelt define a similar hierarchy to that suggested here: pixel-edges-boundary-regions-object-entity[132] while van der Stelt introduces a dimensionality of features: pixel-line-region-texture-time-patient.[139] Note that although most terms are used similarly, the sequence is sometimes changed. In contrast to our definition, texture, for example, is labeled with a higher order.

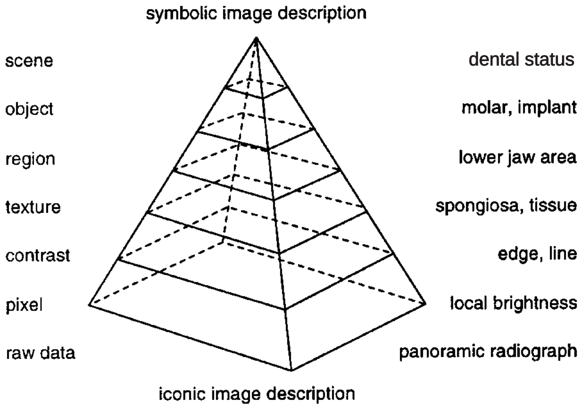

Figure 2 Levels of abstraction of features used for registration (based on [132])

Linkage Image registration is done by linkage of features or feature properties. For example, control points are extracted from external marker positions to establish the correspondence between the reference and subsequent radiographs. 'Exact' coupling of the properties of the features precisely maps them from the first image onto those in the second. Usually, this is done if the number of features exactly equals the number of parameters of the registration model in use.[33–35,38,39,47,49,50,55,62] If the number of features is larger than the number of parameters, 'approximate' linkage is applied to distribute the matching error over all features,[36,37,41,43–45,51,54,56,57,60,63,64] e.g. by a least-squares fit.[120] Furthermore, the linkage of features can be done in 'other' domains than the spatial, e.g. in the logarithmic or frequency (Fourier) domain.[42]

3.1.3 Transform The third criterion for classification of geometry registration procedures is the transform in use. Geometric registration is either model-based, with fixed degrees of freedom, or completely elastic. The latter case equals an infinite degree of freedom, i.e. a transformation model of infinite complexity. Furthermore, registration is performed either on the entire image or on sub-images. Therefore, the transform of geometric *a posteriori registration is characterized by its elasticity, domain, and further, by the search strategy used for determination of its parameters.*

Elasticity The elasticity of transform is differentiated in ascending order of complexity by 'shift' (only translation),[41,56,57] 'rigid body' (shift and rotation),[37,38,54,55] 'RST' (rigid body and scaling),[42,51,58] and 'affine' (RST, regular reflection, and shearing)[35,39,45,47,50,63,64] transforms with two, three, four, and six parameters, respectively (see Glossary for their mathematical definition). A transformation is called affine (sometimes also named planar perspective or weak perspective) if any straight line within the first image is mapped onto a straight line in the second one, preserving the parallelism of any two lines. Note that RST transforms additionally are angle-preserving. The affine transform perfectly describes X-ray imaging with infinite distance between tube and subject which causes the beams to be parallel. Both 'projective'[34,43,44,49,62] and 'bilinear'[33,36] transforms, each of which are characterized by eight parameters, are extensions of affine transformations. However, while projective (perspective) and bilinear transforms likewise map any straight line in the first image onto a straight line in the second, parallelism in general is not preserved (Figure 3). The projective transform perfectly describes X-ray imaging with an infinitely small focal spot size that equals an ideal point-source. 'Other' degrees of freedom or elasticity mostly result in mapping of straight lines onto curves. For example, Webber *et al.* apply a biquadratic transform that is defined by 18 parameters.[60] High-order transforms, for instance, can be applied to model elastic bending of dental films during exposure.

Domain The transform that maps the coordinate system of the reference on to that of the subsequent image can be either local or global. A matching transform is called 'global' if a change in any of the transform parameters effects the entire image. Note that all techniques classified in Section 6.2 of this paper are global. However, Chandermwat *et al.* discuss the feasibility of 'local' transforms for remote sensing applications,[113] while Giachetti applies local registration techniques to determine motion vector fields in ultrasound sequences.[119] Local transforms may vary in granularity from pixel- or voxel-sized to region- or volume-sized for 2D or 3D data, respectively. The domain of a transform can also be regarded as exemplifying the difference between bilinear and projective transforms. Local perspective transforms result in irregularities, while local bilinear transforms are continuous (Figure 3). Therefore, local transforms are mostly based on the bilinear transformation model.

Search strategy The parameter determination of the transformation model for geometric registration may be direct or iterative. 'Direct' determination is based on invariant image features, such as parallelism of lines for affine and less complex transforms. Invariant features allow the straightforward computation of transform parameters.[33–39,41–45,47,49,50,54,56,57,60,62–64] Search-orientated approaches start from one or more initial assumptions and try to find the optimal solution in multiple 'iterative' steps[51,55] guided by some measure of similarity.[12] According to this definition, a brute-force method that evaluates the match for the entire parameter space, i.e. exhaustive search,[41,54,56,57] is also referred to as direct.

3.1.4 Interaction The amount of interaction is important for all clinical applications but difficult to measure. Van der Elsen *et al.* distinguish manual, semi-automatic and automatic algorithms with respect to the requirement of user interaction for the determination of the transform, selection of image properties or neither.[6] In contrast, we define registration methods 'manual' if user interaction is required on both images.[32,40] For example, a method based on corresponding points that are marked interactively in both images[34,35,38,39,43,44,47,49,50,55,60,63] is termed manual. If the user determines landmarks in only one image while the corresponding points are located automatically, the method is referred to as 'semi-automatic'.[33,41,51,54] 'Automatic' methods do not require any involvement in either the reference or subsequent image.[36,37,42,44,45,56,57,62,64] Consequently, most non-automatic methods are classified as manual although the degree of interaction within this category may vary widely.

3.1.5 Examples One of the first 3rd generation algorithms for computer-based *a posteriori* registration of image geometry was introduced by Jeffcoat *et al.* in 1984. Therefore, we will refer to this method as Jeffcoat's approach.[39] Three out of four anatomical landmarks were selected as reference points in non-standardized radiographs: mesial and distal cemento-enamel junctions (CEJ) as well as the radiographic root apices (RRA). Then, the coordinates of the three selected landmarks are used as control points to determine the corresponding projection.[39] Hence, this 2D technique uses intrinsic features on the object level of abstraction that are determined manually in both images. The feature coordinates are mapped exactly to determine directly the parameters of a global affine transform.

The proper categorization of the level of abstraction of particular features is sometimes ambiguous. In particular, it depends on an interpretation of whether

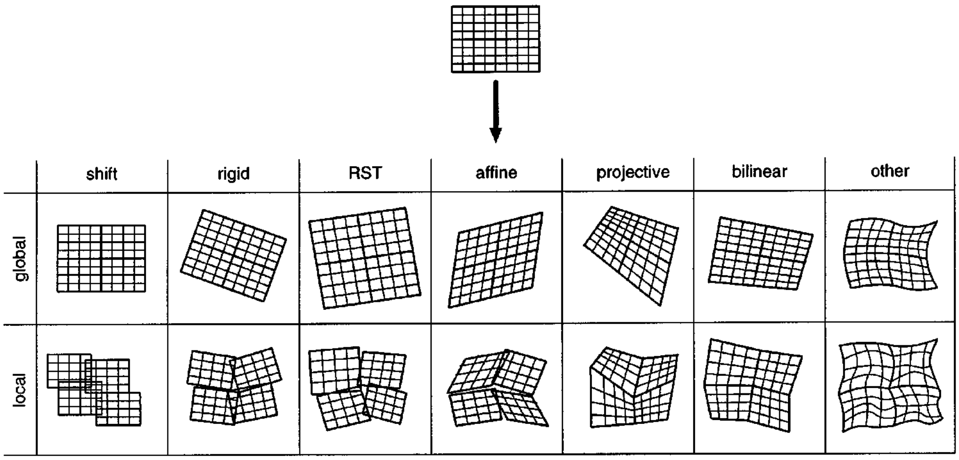

Figure 3 Local and global domains of geometric transforms (based on [6])

a method is based on the pixel or (small) object level. Pixel-based methods often rely on high contrast boundaries of small objects, such as CEJ or RRA. Wenzel uses a large number (up to 16) of manually selected pixels placed on CEJs, RRAs, crown edges, amalgam fillings and other points of high local contrast.[63] These points are distributed evenly within the images. Therefore, this method is clearly based on the pixel level of abstraction. Ettinger and coworkers developed an automatic registration algorithm for subtraction.[36] Boundaries of density gradient are processed to define a characteristic CEJ-profile image that is used as the signature for automatic CEJ detection. This signature includes measures of density changes between the enamel and root region and angles between this interface boundary and the tooth edge. As in the pixel-based approaches the CEJ's coordinates are used for registration. However, this registration procedure undoubtedly is object-based. Note that this method will fail for images without a CEJ, e.g. in dental implants. However, other object-based methods range between these two.[33–35,37,39,62]

3.2 *a posteriori registration of contrast*

Mismatches in X-ray tube settings, quantum noise and film processing, as well as digitization procedures or sensor noise with direct digital modalities,[28] cause gray-level differences. Therefore, quantitative determination from subtraction radiographs requires normalization of the digital image's bitmap.[7,11,15] Direct techniques map the grayscale distribution of one image directly onto the other (Equation 4) while indirect approaches transform both images with respect to a certain reference, e.g., obtained from aluminium references

$$T_{c1}(f_1(x,y)) = T_{c1}(f_1(x',y')) \qquad (5)$$

The advantage of indirect techniques is their absolute normalization, allowing the straightforward determination of quantitative measurements, such as changes in bone volume.[15] However, the errors of both transforms in Equation 5 are additive. Hildebolt and coworkers reported that direct techniques in general are superior, independent of the matching technique used.[11]

Local variations in sensor sensitivity are regarded as a calibration problem rather than a contrast correction task. This agrees with our definitions in Equations 4 and 5. Contrast registration addresses the value range of an image but not its definition. Even though combined methods for calibration and registration in two dimensions have been published,[59] in this review, contrast registration is regarded as a 1D problem of transforming the gray-value histogram. Since, by definition, all techniques for contrast correction are 1D, this criterion is disregarded in the key code (Section 3.3). Therefore, we have evaluated contrast correction methods with respect to the features in use, the transform applied, and the interaction required.

3.2.1 Features As in geometric *a posteriori* registration, the origin of features used for contrast registration is either intrinsic[33,40,41,46,47,52,53] or extrinsic. 'Intrinsic' features for contrast correction are gray values of the pixels themselves. 'Extrinsic' methods are based on an aluminium references such as a slice[59] or step wedge,[31,35,61] or other phantoms with known attenuation coefficients[65] that are placed between the X-ray source and the receptor.

In order to quantify bony changes, Webber *et al.* placed a step wedge next to or behind the jaw to enable contrast correction either for each radiograph independently or of paired radiographs after their subtraction.[61] Using a continuous wedge placed next to the jaw, Allen and Hausmann showed that quantitative measurements are improved if the wedge is used during acquisition of both images to be subtracted.[30] However, the positioning of extrinsic features for contrast registration strongly depends on the application and is therefore not considered in the categorization of methods.

Since the number of features is usually of the same order as the gray-scale, contrast correction methods only link their features approximately, and therefore linkage is not considered further.

3.2.2 Transform In conformity with the elasticity of procedures for geometric registration, contrast registration is either model-based (parametric) or non-parametric. Parametric methods usually rely on either 'linear',[31,47,48,59] 'quadratic',[46,52] or 'cubic'[30,65] polynomials. Webber *et al.* used 'other parametric' functions. They derived a logarithmic function from a simplified absorption equation for mono-energetic X-rays.[61] In all cases, the parameters are determined by pairs of corresponding pixels, e.g. by applying a least squares' fit.[120] 'Non-parametric' transforms directly modify the histogram of the subsequent image in order to relate it as closely as possible to that of the reference image.[35,38,40,41,53] Most of the non-parametric approaches for contrast correction that are included in Section 6.2[35,41] apply the algorithm of Rüttimann *et al.* for contrast correction.[53] Therefore, we refer to this method as Rüttimann's algorithm.

Almost all contrast registration methods operate globally. However, the assumption that both images have the same contents is violated especially in those images with large-sized temporal changes. Therefore, Hildeboldt and coworkers perform a spatial adaptive contrast correction.[38] A masking image is defined which covers those regions with large differences in contrast. Then, the contrast correction is determined only by the unmasked regions but applied to the entire image.[38] Lehmann uses a similar but iterative technique.[43,44] Image regions differing significantly in brightness are segmented automatically after subtraction of the contrast corrected images. These regions are covered for the next iteration of local contrast correction.[44] However, local changes in dental radiology are rather small. Local or iterative contrast correction techniques have only minor effects as compared to global or direct techniques.[43] Therefore, only global contrast transforms are considered below while the search strategy for contrast correction method is not.

3.2.3 Interaction For parametric as well as non-parametric contrast correction methods, pixels within both images must be related to form tuples of gray-values, see Equations 4 and 5. If geometric registration was performed beforehand, this relation is found 'automatically' from the coordinates of the pixels regardless of the domain of contrast features. For extrinsic features, only parts of the images are selected, mostly 'manually',[31] while the entire radiograph is involved if contrast correction is based on intrinsic features. However, Lee and Kim manually select a range of gray-scale to perform additional contrast stretching based on intrinsic features.[41] For the sake of simplicity, only manual *vs* automatic contrast correction methods are differentiated in this category.

3.2.4 Examples Rüttimann's algorithm for retrospective correction of film contrast differences has found wide acceptance in quantitative imaging studies.[11] The cumulative sums of pixels of each gray-scale in the first image are matched to the corresponding cumulative sums of the second image, such that the latter image is less than or equal to that of the first, while also being greater than the next integer digit of the cumulative sum of the first image. Therefore, Rüttimann's algorithm is based on intrinsic features for non-parametric contrast adjustment and is performed automatically, i.e. without manual interaction.[53]

3.3 *Definition of the key code for classification*

Figures 4 and 5 show the relationship between the various criteria for geometry and contrast registration, respectively. Each major criterion for *a posteriori* geometry registration, i.e. dimension, feature, transform, and interaction is abbreviated by its first letter D, F, T, and I, respectively. The hierarchically ordered categories are denoted by digits within brackets. Since the criteria feature and transform have more than one category, those are separated by dots. Each digit reflects the range order of corresponding characteristics where zero denotes not specified (Figure 4). Jeffcoat's approach, for instance, which is based on manually selected intrinsic objects,[39] is coded D(2)F(1.6.1)T(4.2.1)I(1).

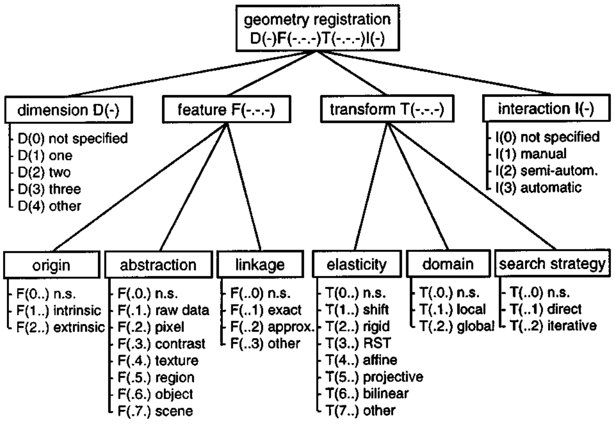

Figure 4 Classification of geometric registration techniques (based on [13]). n.s.: not specified

The same principles are used to code *a posteriori* contrast correction procedures denoted by the capital C (Figure 5). In this terminology, Rüttimann's algorithm for non-parametric automatic contrast correction[53] is abbreviated C(1.5.2).

However, the key code is not restricted to computer-based *a posteriori* registration but is also applicable to 2nd generation approaches for subtraction. For example, Gröndahl's registration method is 2D and based on intrinsic features at the raw data level of abstraction. The features are linked approximately because in general the images will not match exactly for any of the possible positions. Furthermore, the rigid transform is performed globally and iterated manually.[122] Therefore, Gröndahl's procedure for the *a posteriori* alignment of standardized film-based radiographs, registered manually during their digitization, is denoted D(2)F(1.1.2)T(2.2.2)I(1).

All references for computer-based *a posteriori* registration of geometry and contrast that are given in Sections 6.2 and 6.3, respectively, are coded according to this classification.

3.4 *The optimal code for routine applications in dental radiology*

The previous paragraphs have reviewed the various methods for computer-based *a posteriori* registration of both geometry and contrast. The above classification with the format D(-)F(-.-.-)T(-.-.-)I(-)C(-.-.-) is now analysed for its applicability to clinical practice.

The higher the dimensionality, the more information is involved for geometry registration and the more effectively the technique operates. Therefore, robust techniques should be 2D and 3D when applied to 2D and 3D data, respectively. For most dental applications, optimal techniques are coded D(2) or D(3).

Intrinsic features support clinical practicability, reduce costs and allow the inclusion of past image

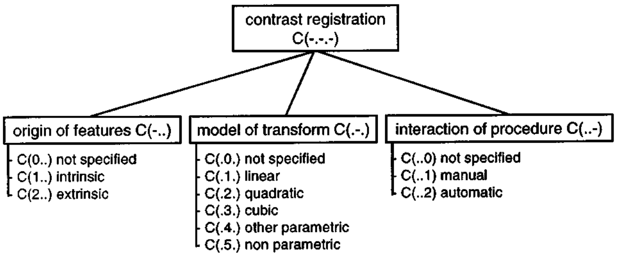

Figure 5 Classification of contrast registration techniques

data in longitudinal studies. Highly abstracted features may result in robust registration. However, the problem of robustness is not solved in general but shifted to the segmentation procedure applied to determine high-level features. Furthermore, the methods will fail for radiographs without any such features. In contrast, features at a lower level of abstraction allow the wider application of the technique. Therefore, the features used should be at the raw data or pixel level of abstraction. Since the approximate linkage of features or feature properties distributes the error, that type of linkage is more robust with single dislocated features which may result from errors during manual or automatic segmentation. Hence, for most clinical applications F(1.1.2) or F(1.2.2) techniques are preferable.

Intraoral radiography has been shown to rely on perspective projection.[13,56,116] Therefore, the transformation model of projective geometry should be used for *a posteriori* registration. Since the radiographs are acquired by a single receptor, the transform is global. A direct search strategy for parameter determination guarantees the best solution while iterative algorithms may be restricted to a local minimum of the feature space. Therefore, the T(5.2.1) techniques may be superior for most clinical applications.

Interaction is time consuming but avoidable using automatic I(3) procedures.

Comparable views hold for contrast correction techniques. Intrinsic features are easier to handle and allow a wider range of applications than extrinsic. In contrast to geometric correction, a precise model for a contrast transform that can be determined theoretically does not exist. Consequently, non-parametric methods are superior to parametric ones.[7,11,15] Again, automatic methods avoid user interaction. In summary, C(1.5.2) techniques such as Rüttimann's algorithm[53] are appropriate for most clinical applications.

In conclusion, the preferable code for *a posteriori* registration for dental applications is D(2/3)F(1.1/2.2) T(5.2.1)I(3)C(1.5.2). It can be seen in Section 6.2 that the only procedure matching this code is the two-stage technique of Lehmann.[44] In this approach, a robust first stage is employed for automatic coarse alignment of RST movements that subsequently allows direct determination of parameters for automatic perspective registration.[44] However, it must be pointed out that this match in code does not necessarily yield the best performance *in vivo*.

4 Clinical applications of 3rd generation subtraction

As it has been reported for 2nd generation subtraction,[2,8,19] computer-based, *a posteriori* registration of geometry and contrast has numerous applications in dental radiology. The literature review below concerns only those papers that refer to a 3rd generation method for geometry registration (Section 3). Computer-based registration of contrast is, of course, already part of numerous 2nd generation systems, and as such reviewed elsewhere.[7,11,15] Although this section is ordered by clinical topic, each technique is classified according to the code in Section 6.3.

- *Technical evaluation*[30,32–39,41–45,48–54,56–58,60–66,69–71,79,97–100]
  Several papers report technical details, methods, and algorithms without reference to specific clinical applications. The major aim is to describe the dependencies and technical parameters of these 3rd generation systems as well as the results and quality of image registration or subtraction. This is true not only for almost all of the papers reviewed in Section 6.2 but also for some of the applications listed in Section 6.3.[66,69–71,79,97–100]
- *Lesions in teeth*[80,90,96,102]
  Computer-based digital subtraction has been applied to the diagnosis and monitoring of dental caries from a longitudinal evaluation of changes in density over time. This group of papers also includes root lesion and other dental pathologies.
- *Periodontal diseases*[31,40,46,47,59,68,72–76,81–85,87,89,91,95,101]
  The evaluation of destructive periodontal diseases[46,47,68,72,74,81–85,87,91,95] or other changes in alveolar bone,[31,40,59,73,75,76,89,101] is an important task in intraoral imaging. Digital subtraction radiography and computer-based registration techniques have been used to monitor the progression of disease and the outcome of treatment.[1,18,20] Studies have been based on either simulated periodontal lesions in dried skulls or cadavers or clinically in patients.
- *Implantology*[93,94]
  Dental implantology has been established over the last decades as a reliable method for replacing lost teeth. The examination of bone quantity and quality for planning implant therapy or during implant healing is usually performed on serial radiographs.[3] In this group of applications, the data consists of either implants or bony changes, or a combination of both.
- *Bone densitometry*[77,78,86,88,92]
  Subtraction-based bone densitometry can be a useful way to estimate total mass changes. However, without linkage to 2D, or ideally 3D, anatomical structures, mass changes can be hard to interpret without extensive remodeling of trabecular architecture. Another potential pitfall for interpreting mass changes based on subtraction radiography is the use of poorly constructed *in vitro* test models. Drilling circular holes in dry bones or placing cortical bone chips in dry or wet jaws does not represent the changes found in periodontal diseases, apical granulomas or osteoporosis. It is possible to perfuse dry bones with epoxy resin soft tissue substitutes to reproduce accurately X-ray absorption and scattering. In addition, bones can be cut into thick serial sections to allow direct access to individual traveculae for minute chemical or mechanical

reduction of the trabecular bone mass to simulate bone remodeling. Finally, by using relocation bolts, the sections can be reassembled into a whole bone for irradiation.[109]

- *Temporomandibular joint function*[55]
  Third generation subtraction systems have been applied to functional evaluation of regular and irregular movements in the temporomandibular joint. In these papers, the TMJ is the only object of interest.
- *Forensic dentistry*[67,103]
  Post-mortem identification of individuals is an essential part of forensic dentistry. Third generation methods of geometric registration are required for the adjustment of images acquired post-mortem with those obtained *in vivo* to determine whether they come from the same individual. Therefore, geometric standardization for *a priori* registration is usually not applicable and hence, forensic applications have not been reported for 1st or 2nd generation systems.

5 Discussion

Today, subtraction radiography is an established modality in dental research but is infrequently used in routine clinical practice.[5,24] In this review, different generations of subtraction systems have been defined (Figure 1). Focusing on 3rd generation approaches, we have introduced a uniform nomenclature and derived a technical code for objective classification of reported techniques. However, this code is also valid for 1st and 2nd generation of subtraction systems. In total, 36 papers defining computer-based *a posteriori* registration have been identified in the literature, covering the entire spectrum of possible codes: 27 of them define geometric registration procedures.

Furthermore, 38 applications of computer-based geometric registration are reported. However, only a few methods are employed routinely in clinical practice. Seventeen papers refer to the D(2) F(1.2.2)T(4.2.1)I(1) computer program developed by Wenzel and coworkers,[63,129] 12 applications use Jeffcoat's algorithm,[39] and at least two with the approach by Ellwood *et al.* have been published.[35] Note that both latter algorithms are identically coded D(2)F(1.6.1)T(4.2.1)I(1).

In Section 3.4 we have pointed out that the D(2/3) F(1.1/2.2)T(5.2.1)I(3)C(1.5.2) code may characterize systems that are superior for routine use. The best performing contrast correction method is intrinsic non-parametric, and automatic and is coded C(1.5.2). In fact, Rüttimann's C(1.5.2)-coded algorithm[44] nowadays is applied to almost all procedures cited in Section 6.3 that include contrast correction. Note that in 2nd generation systems, the C(1.2.2) coded quadratic contrast correction method suggested by Rüttimann *et al.* earlier in 1981 is also often applied.[52] Concerning geometry registration, superior algorithms should run automatically, i.e. they should be I(3)-coded. However, most applications that are based on[35,39,63] use manual registration of geometry. This might be one of the major reasons why subtraction is still seldom used routinely in dentistry.

Another problem that hinders broad adoption of subtraction is the lack in comparability of 3rd generation methods. This problem is addressed by this review only in part. As mentioned previously, the techniques coded D(2/3)F(1.1/2.2)T(5.2.1)I(3)C(1.5.2) will not necessarily be applicable clinically. Systematic evaluation of the robustness[128] or applicability of methods is still lacking.[131] In general, there are several ways to address the correctness and robustness of registration procedures:

(1) *Simulations based on only one image*
The advantage of simulations is that the transform to be corrected by the registration prospectively is known exactly. However, the results obtained by simulations may not be transferable into clinical practice since the artificial situation excludes realistic amounts of noise and other errors.[100]

(2) In vitro *data acquired in adjustable geometry*
In vitro data is acquired from dried cadavers or phantoms. Again, the results obtained *in vitro* might not be transferable directly to routine applications. Furthermore, *a priori* registration and simulated misalignments do not provide the precision needed for high resolution intra-oral radiographs.

(3) *Unmodified* in vivo *data*
Evidently *in vivo* evaluation properly describes the situation in clinical practice, but here the gold standard is unknown.[142] In particular, the gold standard must be established independently of the method under consideration, which complicates evaluation of registration methods. Allen *et al.* have analysed the relationship of texture measurements to the prediction of correct evaluation in subtraction radiography.[104] Ostuni and Dunn have defined a measure called 'registration potential' that allows for quantification of the ability to register two planar transmission images independently of the complexity of the registration model in use.[133] Their work might help to define the gold standard for registering *in vivo* data.

(4) *Manipulated* in vivo *data*
The fourth approach for system evaluation overcomes this problem using manipulated *in vivo* data. West *et al.* describe a method for the retrospective comparison of different image registration techniques.[29] *In vivo* data for brain matching was acquired and fiducial markers used to determine reliably the true movement between the data sets as a gold standard. The marker information was unrecoverably removed from the data sets before they were used for blind evaluation of the accuracy of several *a posteriori* registration techniques. A similarly manipulated *in*

vivo data set might help in the future for objective comparison of 3rd generation subtraction techniques in dentistry.

(5) *Virtual X-ray data*
Simulations based on several images might be suitable for the evaluation of registration procedures if the source of the data is obtained from other modalities such as CT. CT provides a three-dimensional representation of the attenuation coefficients of the objects imaged and enables the computation of virtual X-rays in any geometry. In addition, noise can be added. However, modern spiral CT devices still have insufficient resolution if used for virtual intra-oral imaging compared with direct digital systems. Therefore, tuned aperture computed tomography (TACT) might be a solution.

A wide variety of image matching methods have been proposed for dental applications, but assumption made for these methods differ considerably. The code for classification of 3rd generation subtraction systems, which has been presented in this review, defines a basis for comparability of methods. The code can be extended simply by adding further capitals, e.g. for the classification of computer-based analysis of subtraction images. The post-processing of subtraction images,[121] automated segmentation of lesions and quantitative measurements from the subtraction images[31,108,114,126] are only some examples of computer-aided radiographic analysis[23,24,26] where extension of the classification might be advantageous. Further, it might be useful to extend the code by an additional class describing the interpolation techniques in use.[14,140]

6 References

6.1 *Papers of general interest on image registration and digital subtraction techniques*

The following articles mostly are surveys or reviews of particular interest in the field of image registration for subtraction and digital imaging in dental radiology. They are characterized by a brief summary and the number of references.

1. Benn DK. A review of the reliability of radiographic measurements in estimating alveolar bone changes. *J Clin Periodontol* 1990; **17:** 14-21.
The reliability of radiographic measurements in estimating alveolar bone changes is reviewed critically. Factors affecting a radiographic monitoring system (irradiation geometry, sensing devices, film selection and processing, reference point determination and distance measures, as well as intervals for routine radiographic examinations) are discussed and existing computer systems described. Radiography is seen as a cheap, quick and, within certain limits, accurate method for longitudinal monitoring of alveolar bone loss. However, variations in irradiation geometrics result in different images of identical regions of bone and when comparing sequential films, these artifactual changes could be interpreted as evidence of disease. (30 references)
2. Brägger U. Digital imaging in periodontal radiography. *J Clin Periodontol* 1988; **15:** 551–557.
This review of the early development of digital imaging techniques for the analysis of dental radiographs puts the emphasis on the potential impact on periodontal research. The review covers video-based image processing, qualitative and quantitative subtraction techniques, processing of subtraction images, densitometric analysis, and clinical studies. (68 references)
3. Brägger U. Radiographic parameters for the evaluation of peri-implant tissues. *Periodontology 2000* 1994; **4:** 87–97.
This review summarizes the radiographic parameters currently applied for the evaluation of oral implants (peri-implant bone height and quality, densitometric evaluation, bone mineral content, digital subtraction radiography, and computer-assisted densitometric image analysis CADIA) and critically evaluates each method for its clinical validity and potential for research. (46 references)
4. Brown LG. A Survey of image registration techniques. *ACM Comp Surv* 1992; **24:** 325–376.
This review of geometric registration techniques classifies both the methods (correlation and sequential methods, Fourier methods, point mapping, and model-based matching) and their characteristics (feature space, similarity measure, search space, and search strategy). Examples cover numerous applications in research and technology. (100 references)
5. Duncan JS, Ayache N. Medical image analysis: progress over two decades and the challenges ahead. *IEEE Trans Pattern Analysis and Machine Intelligence* 2000; **22:** 85–106.
This historical review divides the last two decades of medical image analysis into four periods (2D image analysis (pre-1980 to 1984), knowledge-based strategies (1985–1991), mathematical-model-driven approaches (1992–1998), and advanced imaging and realistic visualization (1999 and beyond)). Image registration is referred to as the key feature within all periods. As one of the future challenges, more sophisticated registration algorithms, such as information-theoretical (e.g. mutual information-driven) and/or intensity-based registration have to be incorporated into more of the software packages that are available as part of commercial medical imaging equipment and stand-alone medical image analysis workstations. (139 references)
6. van den Elsen PA, Pol EJD, Viergever MA. Medical image matching: a review with classification. *IEEE Eng Med Biol* 1993; **12:** 26–39.
This review of geometric registration techniques in medicine focuses on 3D applications. The classification criteria (dimensionality, origin of image properties, domain of transform, elasticity of transform, property coupling, parameter determination, and interaction) are subsets of the code introduced in this paper. The literature review (matching on extrinsic properties, matching on intrinsic properties, interactive matching, matching on point pairs, matching on structures, moments or principal axes matching and methods using correlation) covers tomographic imaging modalities, such as CT, MRI, PET, and SPECT, and mono- as well as multi-modal registration applications. (125 references)
7. Fourmousis I, Brägger U, Bürgin W, Tonetti M, Lang NP. Digital image processing: I. Evaluation of grey level correction methods in vitro. *Clin Oral Implants Res* 1994; **5:** 37–47.
The aim of this study was to define an in vitro *model for a priori combined with a posteriori registration. Furthermore, seven methods of gray-level correction based either on a linear least squares approximation (parametric), or on the cumulative density functions (non-parametric), with and without reference structures, were tested. Non-parametric methods were found to be superior while the use of reference structures did not further improve the ability of normalization methods to correct gray-level mismatches between radiographic pairs. (37 references)*
8. Gröndahl K. Computer-assisted subtraction radiography in periodontal diagnosis. *Swedish Dent J* 1987; Supplement **50:** 1–44.

This thesis is based on six papers (a summary of each is included) covering the development, validation, and use of early subtraction techniques in periodontal diagnosis. (88 references)

9. Gröndahl HG. Digital radiology in dental diagnosis: a critical view. *Dentomaxillofac Radiol* 1992; **21:** 198–202.
   Medical history is replete with technologies, which are rapidly accepted but then quickly disappear as their lack of benefit becomes evident. To implement rapidly only those techniques of indisputable value, evaluation must run parallel to development. Digital dental radiography is seen in its infancy but also as basis for promising techniques, i.e. subtraction and tomosynthesis. (11 references)
10. Hausmann E. Digital subtraction radiography: Then (1983) and now (1998). *J Dent Res* 1999; **78:** 7–10.
   This editorial briefly reviews digital subtraction techniques in dental radiology focusing on a priori registration. In particular, the paper covers single-photon absorptiometry, digital radiographic subtraction, alignment systems for serial X-rays suitable for image subtraction, Jeffcoat's approach to evaluation of alignment systems[127] *as well as the development of a model to test realignment systems. (14 references)*
11. Hildebolt CF, Walkup RK, Conover GL, Yokoyama-Crothers N, Bartlett TQ, Vannier MW *et al*. Histogram-matching and histogram-flattening contrast correction methods: A comparison. *Dentomaxillofac Radiol* 1996; **25:** 42–47.
   Direct (histogram flattening) and indirect (histogram matching) approaches for non-parametric contrast correction are discussed and compared experimentally. It is shown that direct methods are superior. (15 references)
12. Lehmann TM, Sovakar A, Schmitt W, Repges R. A comparison of mathematical similarity measures for digital subtraction radiography. *Comp Biol Med* 1997; **27:** 151–167.
   This paper compares eight similarity measures (cross covariance coefficient (CCC), correlation of binary edge images, stochastic sign change, sum of absolute values of the difference, standard deviation of the difference image, edges of difference function, standard deviation of the histogram of the difference image, and entropy of the histogram of the difference image(EHDI)) with respect to resolution, intensity, linearity, and runtime when applied for registration of dental radiographs. EHDI and CCC are found to be the overall best measures. EHDI is preferable whenever the computation time is a critical factor. (28 references)
13. Lehmann TM. *Geometrische Ausrichtung medizinischer Bilder am Beispiel intraoraler Radiographien*. Aachener Informatik-Berichte 1998; no. 9, Fachgruppe Informatik, RWTH Aachen, Germany.
   This thesis (in German) reviews registration techniques in medical imaging, with particular reference to intra-oral radiographs. Novel techniques for automatic registration of geometry are introduced. The criteria applied to the classification of geometry and contrast registration methods are the same as the code used in this paper. (184 references)
14. Lehmann TM, Gönner C, Spitzer K. Survey: Interpolation methods in medical image processing. *IEEE Trans Med Imaging* 1999; **18:** 1049–1075.
   Interpolation is needed whenever a posteriori registration of geometry is applied. This exhaustive survey defines general features (interpolation vs *approximation, DC-constancy* vs *DC-inconstancy) to compare interpolation and approximation techniques (truncated and windowed sinc, nearest neighbor, linear, quadratic, cubic B-spline, cubic, Lagrange, and Gaussian) on different scales with respect to spatial and Fourier properties, computational complexity, runtime evaluation, and qualitative and quantitative interpolation error. The cubic* 6×6 *interpolator with continuous second order derivative is recommended for most interpolation tasks. (37 references)*
15. Likar B, Pernus F. Evaluation of three contrast correction methods for digital subtraction in dental radiology: An *in vitro* study. *Med Phys* 1997; **24:** 299–307.
   This in vitro *study compares the cumulative density function (CDF,*[53] *code: C(1.5.-)), the optical density thickness function (ODSF, also referred to as aluminum equivalent thickness,*[59] *code: C(2.1.-), and the least square quadratic approximation (LSQA,*[46] *code: C(1.2.-)) for contrast correction in dental radiographs. Automatic* vs *manual implementation is disregarded. The authors conclude that ODSF is preferable because it allows for the estimation of changes in bone volumes. (19 references)*
16. Maintz JBA, Viergever MA. A survey of medical image registration. *Med Image Anal* 1998; **2:** 1–36.
   This paper presents an exhaustive survey of recent (published 1993 or later) publications concerning medical image registration techniques. The publications are classified according to nine salient criteria (dimensionality, basis, name and domain of transform, interaction, optimization, modalities, subject, and object), the main dichotomy of which is extrinsic vs *intrinsic methods. Objects are classified by head, thorax, abdomen, pelvis and perineum, limbs, and spine and vertebrae. However, the review focuses on multi-modal matching of brain images and neglects dental applications. It concludes that the bulk of interesting intrinsic methods is based on either segmented points or surfaces, or on techniques using the entire images. (305 references)*
17. Meijering EH, Niessen WJ, Viergever MA. Retrospective motion correction in digital subtraction angiography: A review. *IEEE Trans Med Imaging* 1999; **18:** 2–21.
   This paper reviews the field of a posteriori registration for digital subtraction angiography. Motion artifacts are classified as patient- or acquisition-related. Motion correction algorithms are ordered by complexity of the transform, similarity measure, sub-pixel precision, optimization and acceleration, and gray-level distortion correction. Furthermore, the review discusses control-point selection and displacement interpolations, comparisons of similarity measures, interpolation techniques for sub-pixel precision, optimization strategies and related issues, as well as multiplicative vs *additive gray-level distortion models. (171 references)*
18. Reddy MS. Radiographic methods in the evaluation of periodontal therapy. *J Periodontol* 1992; **63:** 1078–1084.
   This review describes a priori and a posteriori methods available for periodontal clinical trials by concentrating on the major errors (geometric distortion and radiographic contrast) that limit the ability of radiographic methods to measure bone loss with respect to bone height (1D), area (2D), and volume (3D). The paper discusses methods for controlling sources of errors, direct vs *indirect digital imaging and sequential analyses such as subtraction. (33 references)*
19. Reddy MS, Jeffcoat MK. Digital subtraction radiography. *Dent Clin North Am* 1993; **37:** 553–565.
   This review of digital subtraction radiography focuses on clinical applications with respect to periodontal diseases, caries diagnosis, periapical lesions after root canal therapy, implant dentistry and periimplant defects rather than on imaging methods. The review discusses the history of subtraction radiology, computerized digital imaging for subtraction, validation of digital subtraction radiography, and clinical application of digital subtraction radiography. (48 references)
20. Reddy MS, Jeffcoat MK. Methods of assessing periodontal regeneration. *Periodontol 2000* 1999; **19:** 87–103.
   This review describes the assessment of regenerative technology and the comparison of different modes of therapy by means of histology, direct measurement of bone, periodontal probing and radiographic analyses, such as conventional, direct digital, and digital subtraction radiography. Only newer methods, such as digital subtraction radiography, provide the degree of precision needed to detect small differences between different treatment modalities. (102 references)
21. Samarabandu J, Allen K, Hausmann E, Acharya R. Registration techniques for digital subtraction radiography. *Dentomaxillofac Radiol* 1994; **23:** 117–119.
   This Letter to the Editor discusses the paper by Dunn et al.[34] *and includes the addressed authors' response. Similar to the terminology used in our review, registration is defined as simple alignment (geometry and contrast) of a pair of radiographs prior to subtraction. Furthermore, the authors differentiate between*

techniques to stabilize projection geometry (a priori registration) and those used to register prior to subtraction (a posteriori registration). (5 and 4 references in the letter and response, respectively)

22. van der Stelt PF. Improved diagnosis with digital radiography: Editorial review. *Curr Opin Dent* 1992; **2:** 1 – 6.
This editorial briefly reviews computer technology for radiography in medicine and dentistry. It covers digital technologies for image acquisition, subtraction radiography, image reconstruction, and digitized image interpretation. Papers of particular interest, published within the period of review (1990/91), are ranked by special or outstanding interest. Additionally, selected papers are summarized. (51 references)
23. van der Stelt PF. Computer-assisted interpretation in radiographic diagnosis. *Dent Clin North Am* 1993; **37:** 683 – 696.
This review covers the application of computer-aided procedures to the analysis of digital radiographs with respect to improving the quality of radiographic diagnosis. Image processing is applied to improve image quality and extract features from digital radiographs. Computer programs are used to integrate image and patient data originating from different sources. The aim of these procedures is to support the clinician in the decision-making process. (31 references)
24. Vandre RH, Webber RL. Future trends in dental radiology. *Oral Surg Oral Med Oral Pathol Oral Radiol Endod* 1995; **80:** 471 – 478.
This paper addresses current technology and future trends in dental radiology. It focuses on digital subtraction radiography (methods of a priori and a posteriori registration and contrast adjustment as well as examples of clinical applications), tuned aperture computed tomography (principles and potentials for clinical application), and computer aided diagnosis (application of pattern recognition and artificial intelligence to dental radiographs). (46 references)
25. Versteeg CH, Sanderink GC, van der Stelt PF. Efficacy of digital intra-oral radiography in clinical dentistry. *J Dent* 1997; **25:** 215 – 224.
This review emphasizes the comparison of intra-oral digital with film-based imaging and concludes that digital imaging certainly has great potential, especially with respect to improvement of diagnostic quality and automated image analysis. Other subjects addressed in this paper are image quality, image acquisition, image manipulation and the application software as well as its contribution to systems efficacy. (63 references)
26. Wenzel A. Influence of computerized information technologies on image quality in dental radiographs. *Tandlaegebladet* 1991; **95:** 527 – 559.
This thesis is based on nine papers (a summary of each is included) covering recording (digital recording, matrix resolution, bit depth, computer-assisted instructions of radiographic recording techniques) as well as manipulation (contrast enhancement, edge enhancement, subtraction) of radiographs aided by computer technologies. Subtraction programs for computer-based a posteriori registration are addressed. (300 references)
27. Wenzel A. Computer-aided image manipulation of intraoral radiographs to enhance diagnosis in dental practice: A review. *Int Dent J* 1993; **43:** 99 – 108.
This paper reviews image processing techniques applied to dental radiology and covers contrast enhancement, edge enhancement, and digital subtraction. (96 references)
28. Wenzel A, Grondahl HG. Direct digital radiography in the dental office. *Int Dent J* 1995; **45:** 27 – 34. (erratum **45:**391).
Direct digital imaging has significantly reinforced the establishment of digital subtraction in dental radiology. This paper compares the then available direct digital X-ray systems for general dental practice in respect of system configuration, system resolution, noise, software, and user interface. The authors discuss their advantages and disadvantages. In conclusion, direct digital imaging is seen to be in many ways still in its infancy, with rapid technological development likely. (49 references)
29. West J, Fitzpatrick JM, Wang MY, Dawant BM, Maurer CR, Kessler RM. Comparison and evaluation of retrospective intermodality image registration techniques. *Proc SPIE* 1996; **2710:** 332 – 347.
This paper describes a project whose principal goal was to use a prospective method based on fiducial markers as a 'gold standard' to perform an objective, blind evaluation of the accuracy of several retrospective image-to-image registration techniques. Eleven 3D registration techniques were applied and compared. The paper presents preliminary results of this study along with a brief description of each registration technique and estimate both preparation and execution time needed to perform the registration. (23 references)
Project web homepage: http://cswww.vuse.vanderbilt.edu/~image/registration/

6.2 *Definition of computer-based registration procedures*

The following papers focus on the methodology of registration procedures for either geometry or contrast adjustment or both. They are taken exclusively from the field of dental radiology. For a review of their various applications for *a posteriori* registration prior to subtraction see the next section.

30. Allen KM, Hausmann E. Analytical methodology in quantitative digital subtraction radiography: Analyses of the aluminum reference wedge. *J Periodontol* 1996; **67:** 1317 – 1321.

    D(–)F(–.–.–)T(–.–.–)I(–)C(2.3.0)

31. Brägger U, Pasquali L, Rylander H, Carnes D, Kornman KS. Computer-assisted densitometric image analysis in periodontal radiography. A methodological study. *J Clin Periodontol* 1988; **15:** 27 – 37.

    D(–)F(–.–.–)T(–.–.–)I(–)C(2.1.1)

32. Brocklebank LM, McGovern C, Jin J, Siebert JP, van der Stelt PF. A flexible reconstruction program for use in digital subtraction radiology. *J Dent Res* 1999; **78:** 535 (abstr.).

    D(2)F(1.2.0)T(0.2.0)I(1)C(–.–.–)

33. Byrd V, Mayfield-Donahoo T, Reddy MS, Jeffcoat MK. Semi-automated image registration for digital subtraction radiography. *Oral Surg Oral Med Oral Pathol Oral Radiol Endod* 1998; **85:** 473 – 478.

    D(2)F(1.6.1)T(6.2.1)I(2)C(–.–.–)

34. Dunn SM, van der Stelt PF, Ponce A, Fenesy K, Shah S. A comparison of two registration techniques for digital subtraction radiography. *Dentomaxillofac Radiol* 1993; **22:** 77 – 80.

    D(2)F(1.6.1)T(5.2.1)I(1)C(–.–.–)

35. Ellwood RP, Davies RM, Worthington HV. Evaluation of a dental subtraction radiography system. *J Periodontal Res* 1997; **32:** 241 – 248.

    D(2)F(1.6.1)T(4.2.1)I(1)C(2.5.2) application of [53] for contrast correction combined with aluminum step-wedge

36. Ettinger GJ, Gordon GG, Goodson JM, Socransky SS, Williams R. Development of automated registration algo-

rithms for subtraction radiography. *J Clin Periodontol* 1994; **21:** 540 – 543.

D(2)F(1.6.2)T(6.2.1)I(3)C(–.–.–) after coarse manual alignment

37. Fisher E, van der Stelt PF, Dunn SM. 3D registration of surfaces for change detection in medical images. *Proc SPIE* 1997; **3034:** 599 – 610.

D(3)F(1.6.2)T(2.2.1)I(3)C(–.–.–)

38. Hildebolt CF, Brunsden B, Yokoyama-Crothers N, Pilgram TK, Townsend KE, Vannier MW *et al.* Comparison of reliability of manual and computer-intensive methods for radiodensity measures of alveolar bone loss. *Dentomaxillofac Radiol* 1998; **27:** 245 – 250.

D(2)F(1.2.1)T(2.2.1)I(1)C(1.5.2)
excluding differing regions from contrast correction

39. Jeffcoat MK, Jeffcoat RL, Williams RC. A new method for the comparison of bone loss measurements on non-standardized radiographs. *J Periodontal Res* 1984; **19:** 434 – 440.

D(2)F(1.6.1)T(4.2.1)I(1)C(–.–.–)

40. Leder AJ, Simon BI, Deasy M, Fenesy KE, Dunn S. Histological, clinical, and digital subtraction radiographic evaluation of repair of periodontal defects resulting from mechanical perforation of the chamber floor using ePTFE membranes. *Periodontal Clin Investig* 1997; **19:** 9 – 15.

D(2)F(1.0.0)T(0.2.0)I(1)C(1.5.1)

41. Lee SH, Kim EK. Development of subtraction radiography system by using personal computer. Proceedings of the 10th International Congress of Dento-Maxillo-Facial Radiology; Seoul, Korea 1994; 371 – 376.

D(2)F(1.3.2)T(1.2.2)I(2)C(1.5.1)
manual application of [53] for contrast correction

42. Lehmann TM, Goerke C, Schmitt W, Kaupp A, Repges R. Rotation-extended cepstrum technique optimized by systemic analysis of various sets of x-ray images *Proc SPIE* 1996; **2710:** 390 – 401.

D(2)F(1.1.3)T(3.2.1)I(3)C(–.–.–)

43. Lehmann TM, Gröndahl K, Gröndahl HG, Schmitt W, Spitzer K. Observer-independent registration of perspective projection prior to subtraction of in vivo radiographs. *Dentomaxillofac Radiol* 1998; **27:** 140 – 150.

D(2)F(1.2.2)T(5.2.1)I(1)C(–.–.–)

44. Lehmann TM. A two-stage algorithm for model-based registration of medical images. *Proceedings of the International Conference on Pattern Recognition ICPR'98, Brisbane, Australia,* 1998; **1:** 344 – 351.

D(2)F(1.1.3)T(3.2.1)I(3)C(–.–.–) followed by
D(2)F(1.2.2)T (5.2.1)I(3)C(1.5.2) application of [42] for 1st stage geometry and [53] for contrast correction

45. Likar B, Pernus F. Automatic extraction of corresponding points for the registration of medical images. *Med Phys,* 1999; **26:** 1678 – 1686.

D(2)F(1.2.2)T(4.2.1)I(3)C(–.–.–)

46. Ohki M, Okano T, Yamada N. A contrast-correction method for digital subtraction radiography. *J Periodontal Res* 1988; **23:** 277 – 280.

D(–)F(–.–.–)T(–.–.–)I(–)C(1.2.2)

47. Orstavik D, Farrants G, Wahl T, Kerekes K. Image analysis of endodontic radiographs: digital subtraction and quantitative densitometry. *Endod Dent Traumatol* 1990; **6:** 6 – 11.

D(2)F(1.2.1)T(4.2.1)I(1)C(1.1.2)

48. Ortman LF, Dunford R, McHenry K, Hausmann E. Subtraction radiography and computer assisted densitometric analyses of standardized radiographs. A comparison study with ^{125}I absorptiometry. *J Periodontal Res* 1985; **20:** 644 – 651.

D(–)F(–.–.–)T(–.–.–)I(–)C(2.1.0)

49. Ostuni J, Fisher E, van der Stelt P, Dunn S. Registration of dental radiographs using projective geometry. *Dentomaxillofac Radiol* 1993; **22:** 199 – 203.

D(2)F(1.2.1)T(5.2.1)I(1)C(–.–.–)

50. Ostuni J, Dunn S. Fast image registration based upon projective planar geometry. *Proc 19th IEEE Annual Northeast Bioengineering Conference. New York, NY, USA.* 1993; 174 – 175.

D(2)F(1.2.1)T(4.2.1)I(1)C(–.–.–)

51. Papika S, Paulsen HU, Shi XQ, Welander U, Linder-Aronson S. Orthodontic application of color image addition to visualize differences between sequential radiographs. *Am J Orthodont Dentofac Orthoped* 1999; **115:** 488 – 493.

D(2)F(1.1.2)T(3.2.2)I(2)C(1.5.2)
application of [53] for contrast correction

52. Ruttimann UE, Okano T, Gröndahl HG, Gröndahl K, Webber RL. Exposure geometry and film contrast differences as bases for incomplete cancellation of irrelevant structures in dental subtraction radiography. *Proc SPIE* 1981; **314:** 372 – 377.

D(–)F(–.–.–)T(–.–.–)I(–)C(1.2.2)

53. Ruttimann UE, Webber RL, Schmidt E. A robust digital method for film contrast correction in subtraction radiography. *J Periodontal Res* 1986; **21:** 486 – 495.

D(–)F(–.–.–)T(–.–.–)I(–)C(1.5.2)

54. Samarabandu J, Allen KM, Hausmann E, Acharya R. Algorithm for the automated alignment of radiographs for image subtraction. *Oral Surg Oral Med Oral Pathol* 1994; **77:** 75 – 79.

D(2)F(1.5.2)T(2.2.1)I(2)C(–.–.–)

55. Sato H, Ohki M, Kitamori H. A method for quantifying positional change of the condyle on lateral tomograms by means of digital subtraction. *J Oral Rehabil* 1998; **25:** 448 – 455.

D(2)F(1.2.1)T(2.2.2)I(1)C(–.–.–)

56. Schmitt W, Lehmann TM. Digitale Radiographie und digitale Bildverarbeitung in der implantologischen Diagnostik. *Z Zahnärztl Implantol* 1993; **9:** 284 – 287.

D(2)F(1.3.2)T(1.2.1)I(3)C(–.–.–)

57. van der Stelt PF, Ruttimann UE, Webber RL. Determination of projections for subtraction radiography based on image

similarity measurements. *Dentomaxillofac Radiol* 1989; **18:** 113–117.

D(2)F(1.1.2)T(1.2.1)I(3)C(–.–.–)

58. Vannier MW, Hildebolt CF, Conover G, Knapp RH, Yokoyama-Crothers N, Wang G. Three-dimensional dental imaging by spiral CT. *Proc SPIE* 1995; **2434:** 346–360.

    D(3)F(1.3.0)T(3.2.0)I(0)C(–.–.–)

59. Vos MH, Janssen PTM, van Aken J, Heethaar. Quantitative measurement of periodontal bone changes by digital subtraction. *J Periodont Res* 1986; **21:** 583–591.

    D(–)F(–.–.–)T(–.–.–)I(–)C(2.1.2)

60. Webber RL, Ruttimann UE, Groenhuis RA. Computer correction of projective distortions in dental radiographs. *J Dent Res* 1984; **63:** 1032–1036.

    D(2)F(2.2.2)T(7.2.1)I(1)C(–.–.–)

61. Webber RL, Ruttimann UE, Heaven TJ. Calibration errors in digital subtraction radiography. *J Periodontal Res* 1990; **25:** 268–275.

    D(–)F(–.–.–)T(–.–.–)I(–)C(2.4.0)

62. Webber RL, Bettermann W. A method for correcting for errors produced by variable magnification in three-dimensional tuned-aperture computed tomography. *Dentomaxillofac Radiol* 1999; **28:** 305–310.

    D(2)F(2.6.1)T(5.2.1)I(3)C(–.–.–)

63. Wenzel A. Effect of manual compared with reference point superimposition on image quality in digital subtraction radiography. *Dentomaxillofac Radiol* 1989; **18:** 145–150.

    D(2)F(1.2.2)T(4.2.1)I(1)C(1.5.2)
    application of [53] for contrast correction

64. Yoon DC. A new method for the automated alignment of dental radiographs for digital subtraction radiography. *Dentomaxillofac Radiol* 2000; **29:** 11–19.

    D(2)F(1.3.2)T(4.2.1)I(3)C(1.5.2)
    application of [53] for contrast correction

65. Yoshioka T, Kobayashi C, Suda H, Sasaki T. Quantitative subtraction with direct digital dental radiography. *Dentomaxillofac Radiol* 1997; **26:** 286–294.

    D(–)F(–.–.–)T(–.–.–)I(–)C(2.3.2)

6.3 *Application of computer-based registration in dental radiology*

The following papers use *a posteriori* techniques that have been defined in the previous section for either geometry or contrast registration or both.

66. Aagaard E, Donslund C, Wenzel A, Sewerin I. Performance for obtaining maximal gain from a program for digital subtraction radiography. *Scand J Dent Res* 1991; **99:** 166–172.

    D(2)F(1.2.2)T(4.2.1)I(1)C(–.–.–) application of [63]
    without contrast registration

67. Andersen L, Wenzel A. Individual identification by means of conventional bitewing film and subtraction radiography. *Forensic Sci Int* 1995; **72:** 55–64.

    D(2)F(1.2.2)T(4.2.1)I(1)C(1.5.2) application of [63] and [53]

68. Armitage GC, Jeffcoat MK, Chadwick DE, Taggart Jr EJ, Numabe Y, Landis JR *et al*. Longitudinal evaluation of elastase as a marker for the progression of periodontitis. *J Periodontol* 1994; **65:** 120–128.

    D(2)F(1.6.1)T(4.2.1)I(1)C(1.5.2) application of [39] and [53]

69. Brettle DS, Ellwood R, Davies R. Determination of the optimal conditions for dental subtraction radiography using a storage phosphor system. *Dentomaxillofac Radiol* 1999; **28:** 1–5.

    D(2)F(1.6.1)T(4.2.1)I(1)C(–.–.–) application of [35] using special step wedge

70. Brocklebank LM, van der Stelt PF, Beattie S, Siebert JP. Reference region selection for density standardisation in digital subtraction radiology. *J Dent Res* 2000; **79:** 601(abstr.).

    D(2)F(1.2.0)T(0.2.0)I(1)C(–.–.–) application of [32]

71. Burdea GC, Dunn SM, Levy G. Evaluation of robot-based registration for subtraction radiography. *Med Image Anal* 1999; **3:** 265–274.

    D(2)F(1.2.1)T(5.2.1)I(1)C(0.0.0) application of[49]
    with unspecified contrast registration

72. Christgau M, Schmalz G, Reich E, Wenzel A. Clinical and radiographical split-mouth-study on resorbable versus non-resorbable GTR-membranes. *J Clin Periodontol* 1995; **22:** 306–315.

    D(2)F(1.2.2)T(4.2.1)I(1)C(1.5.2) application of [63] and [53]

73. Christgau M, Wenzel A, Hiller KA, Schmalz G. Quantitative digital subtraction radiography for assessment of bone density changes following periodontal guided tissue regeneration. *Dentomaxillofac Radiol* 1996; **25:** 25–33.

    D(2)F(1.2.2)T(4.2.1)I(1)C(1.5.2) application of [63] and [53]

74. Christgau M, Bader N, Schmalz G, Hiller KA, Wenzel A. Postoperative exposure of bioresorbable GTR membranes: Effect on healing results. *Clin Oral Investig* 1997; **1:** 109–118.

    D(2)F(1.2.2)T(4.2.1)I(1)C(1.5.2) application of [63] and [53]

75. Christgau M, Schmalz G, Wenzel A, Hiller KA. Periodontal regeneration of intrabony defects with resorbable and non-resorbable membranes: 30-month results. *J Clin Periodontol* 1997; **24:** 17–27.

    D(2)F(1.2.2)T(4.2.1)I(1)C(1.5.2) application of [63] and [53]

76. Christgau M, Bader N, Schmalz G, Hiller KA, Wenzel A. GTR therapy of intrabony defects using 2 different bioresorbable membranes: 12-month results. *J Clin Periodontol* 1998; **25:** 499–509.

    D(2)F(1.2.2)T(4.2.1)I(1)C(–.–.–) application of [63]
    without contrast registration

77. Christgau M, Hiller KA, Schmalz G, Kolbeck C, Wenzel A. Accuracy of quantitative digital subtraction radiography for determining changes in calcium mass in mandibular bone: An *in vitro* study. *J Periodontal Res* 1998; **33:** 138 – 149.

    D(2)F(1.2.2)T(4.2.1)I(1)C(1.5.2) application of [63] and [53]

78. Christgau M, Hiller KA, Schmalz G, Kolbeck C, Wenzel A. Quantitative digital subtraction radiography for the determination of small changes in bone thickness: An *in vitro* study. *Oral Surg Oral Med Oral Pathol Oral Radiol Endod* 1998; **85:** 462 – 472.

    D(2)F(1.2.2)T(4.2.1)I(1)C(1.5.2) application of [63] and [53]

79. Fisher E, van der Stelt PF, Ostuni J, Dunn SM. The effect of independent film and object rotation on projective geometric standardization of dental radiographs. *Dentomaxillofac Radiol* 1995; **24:** 5 – 12.

    D(2)F(2.6.1)T(5.2.1)I(1)C(–.–.–) application of [34]
    using wire frame

80. Hintze H, Wenzel A, Andreasen FM, Swerin I. Digital subtraction radiography for assessment of simulated root resorption cavities. Performance of conventional and reverse contrast modes. *Endod Dent Traumatol* 1992; **8:** 149 – 154.

    D(2)F(1.2.2)T(4.2.1)I(1)C(1.5.2) application of [63] and [53]

81. Jeffcoat MK, Williams RC, Reddy MS, English R, Goldhaber P. Flurbiprofen treatment of human periodontitis: Effect on alveolar bone height and metabolism. *J Periodontal Res* 1988; **23:** 381 – 385.

    D(2)F(1.6.1)T(4.2.1)I(1)C(1.5.2) application of [39] and [53]

82. Jeffcoat MK. Assessment of periodontal disease progression: Application of new technology to conventional tools. *Periodontal Case Rep* 1989; **11:** 8 – 12.

    D(2)F(1.6.1)T(4.2.1)I(1)C(1.5.2) application of [39] and [53]

83. Jeffcoat MK, Page R, Reddy M, Wannawisute A, Waite P, Palcanis K *et al*. Use of digital radiography to demonstrate the potential of naproxen as an adjunct in the treatment of rapidly progressive periodontitis. *J Periodontal Res* 1991; **26:** 415 – 421.

    D(2)F(1.6.1)T(4.2.1)I(1)C(1.5.2) application of [39] and [53]

84. Jeffcoat MK. Radiographic methods for the detection of progressive alveolar bone loss. *J Periodontol* 1992; **63:** 367 – 372.

    D(2)F(1.6.1)T(4.2.1)I(1)C(1.5.2) application of [39] and [53]

85. Jeffcoat MK, Reddy MS, Moreland LW, Koopman WJ. Effects of nonsteroidal antiinflammatory drugs on bone loss in chronic inflammatory disease. *Ann N Y Acad Sci* 1993; **696:** 292 – 302.

    D(2)F(1.6.1)T(4.2.1)I(1)C(1.5.2) application of [39] and [53]

86. Jensen J, Kragskov J, Wenzel A, Sindet-Pedersen S. *In vitro* analysis of the accuracy of subtraction radiography and computed tomography scanning for determination of bone graft volume. *J Oral Maxillofac Surg* 1998; *56:* 743 – 748.

    D(2)F(1.2.2)T(4.2.1)I(1)C(1.5.2) application of [63] and [53]

87. Li KL, Vogel R, Jeffcoat MK, Alfano MC, Smith MA, Collins JG *et al*. The effect of ketoprofen creams on periodontal disease in rhesus monkeys. *J Periodontal Res* 1996; **31:** 525 – 532.

    D(2)F(1.6.1)T(4.2.1)I(1)C(1.5.2) application of [39] and [53]

88. Loftin R, Webber R, Horton R, Tyndall D, Moriarty J. Effect of projective aspects variations on estimates of changes in bone mass using digital subtraction radiography. *J Periodontal Res* 1998; **33:** 352 – 358.

    D(2)F(2.6.1)T(5.2.1)I(3)C(1.2.2) application of [62] and [52]

89. Palcanis KG, Larjava IK, Wells BR, Suggs KA, Landis JR, Chadwick DE *et al*. Elastase as an indicator of periodontal disease progression. *J Periodontol* 1992; **63:** 237 – 242.

    D(2)F(1.6.1)T(4.2.1)I(1)C(1.5.2) application of [39] and [53]

90. Perona G, Wenzel A. Radiographic evaluation of the effect of orthodontic retraction on the root of the maxillary canine. *Dentomaxillofac Radiol* 1996; **25:** 179 – 185.

    D(2)F(1.2.2)T(4.2.1)I(1)C(–.–.–) application of [63]
    without contrast registration

91. Preshaw PM, Geatch DR, Lauffart B, Jeffcoat MK, Taylor JJ, Heasman PA. Longitudinal changes in TCRB variable gene expression and markers of gingival inflammation in experimental gingivitis. *J Clin Periodontol* 1998; **25:** 774 – 780.

    D(2)F(1.6.1)T(4.2.1)I(1)C(1.5.2) application of [39] and [53]

92. Rawlinson A, Ellwood RP, Davies RM. An *in vitro* evaluation of a dental subtraction radiography system using bone chips on dried human mandibles. *J Clin Periodontol* 1999; **26:** 138 – 142.

    D(2)F(1.6.1)T(4.2.1)I(1)C(2.5.2) application of [35]
    using bite block and step wedge

93. Reddy MS, Jeffcoat MK, Richardson RC. Assessment of adjunctive flurbiprofen therapy in root-form implant healing with digital subtraction radiography. *J Oral Implantol* 1990; **16:** 272 – 276.

    D(2)F(1.6.1)T(4.2.1)I(1)C(1.5.2) application of [39] and [53]

94. Reddy MS, Mayfield-Donahoo TL, Jeffcoat MK. A semi-automated computer-assisted method for measuring bone loss adjacent to dental implants. *Clin Oral Implants Res* 1992; **3:** 28 – 31.

    D(2)F(1.6.1)T(4.2.1)I(1)C(–.–.–) application of [39]

95. Reddy MS, Palcanis KG, Barnett ML, Haigh S, Charles CH, Jeffcoat MK. Efficacy of meclofenamate sodium (meclomen) in the treatment of rapidly progressive periodontitis. *J Clin Periodontol* 1993; **20:** 635 – 640.

    D(2)F(1.6.1)T(4.2.1)I(1)C(1.5.2) application of [39] and [53]

96. Reukers E, Sanderink G, Kuijpers-Jagtman AM, van't Hof M. Assessment of apical root resorption using digital reconstruction. *Dentomaxillofac Radiol* 1998; **27:** 25 – 29.

    D(2)F(1.6.1)T(5.2.1)I(3)C(1.5.2) application of [34] and [53]
    (EMAGO)

97. Sander L, Wenzel A, Hintze H, Karring T. Image homogeneity and recording reproducibility with 2 techniques for serial intraoral radiography. *J Periodontol* 1996; **67:** 1288 – 1291.

    D(2)F(1.2.2)T(4.2.1)I(1)C(1.5.2) application of [63] and [53]

98. Shi XQ, Eklund I, Tronje G, Welander U, Stamatakis HC, Engstrom PE *et al*. Comparison of observer reliability in assessing alveolar bone changes from color-coded with

subtraction radiographs. *Dentomaxillofac Radiol* 1999; **28:** 31–36.

D(2)F(1.1.2)T(3.2.2)I(1)C(1.5.2) application of [51] and [53]

99. Vannier MW, Hildebolt CF, Conover G, Knapp RH, Yokoyama-Crothers N, Wang G. Three-dimensional dental imaging by spiral CT. A progress report. *Oral Surg Oral Med Oral Pathol Oral Radiol Endodont* 1997; **84:** 561–570.

D(3)F(1.3.0)T(3.2.0)I(3)C(–.–.–) application of [58]

100. Wenzel A, Sewerin I. Sources of noise in digital subtraction radiography. *Oral Surg Oral Med Oral Pathol* 1991; **71:** 503–508.

D(2)F(1.2.2)T(4.2.1)I(1)C(–.–.–) application of [63]
without contrast registration

101. Wenzel A, Warrer K, Karring T. Digital subtraction radiography in assessing bone changes in periodontal defects following guided tissue regeneration. *J Clin Periodontol* 1992; **19:** 208–213.

D(2)F(1.2.2)T(4.2.1)I(1)C(1.5.2) application of [63] and [53]

102. Wenzel A, Halse A. Digital subtraction radiography after stannous fluoride treatment for occlusal caries diagnosis. *Oral Surg Oral Med Oral Pathol* 1992; **74:** 824–828.

D(2)F(1.2.2)T(4.2.1)I(1)C(–.–.–) application of [63]
without contrast registration

103. Wenzel A, Andersen L. A quantitative analysis of subtraction images based on bite-wing radiographs for simulated victim identification in forensic dentistry. *J Forensic Odontostomatol* 1994; **12:** 1–5.

D(2)F(1.2.2)T(4.2.1)I(1)C(–.–.–) application of [63]
without contrast registration

6.4 *Other papers of interest*

104. Allen K, Emrich L, Piedmonte M, Hausmann E. Relationship of texture measurements to the prediction of correct evaluations in subtraction radiography. *J Periodontal Res* 1992; **27:** 197–206.
(texture analysis in subtraction images is used to differentiate temporal changes and artifacts caused by misalignment)
105. Allen KM, Hausmann E, Kutlubay ME, Loza J, Carpio LC, Ortman L *et al*. Studies of the angular reproducibility of positioning patients adjacent to an x-ray tube: 1. Stent-rod based and extra-oral systems. *J Periodontal Res* 1994; **29:** 174–178.
(precision of stent-based repositioning after 6 months and that of an extra-oral system based on ear rods after 1 month is about 2°)
106. Araki K, Kitamori H, Yoshiura K, Okuda H, Ohki M. Standardized lateral oblique projection of the mandible for digital subtraction radiography. *Dentomaxillofac Radiol* 1992; **21:** 88–92.
(device for a priori registration of lateral oblique projections of the mandible)
107. Baxes GA. *Digital image processing: Principles and applications*. New York: Wiley, 1994.
(non-mathematical textbook with a glossary of technical terms and phrases)
108. Benn DK. Limitations of the digital image subtraction technique in assessing alveolar bone crest changes due to misalignment errors during image capture. *Dentomaxillofac Radiol* 1990; **19:** 97–104.
(quantitative measurements in subtraction images and their evaluation by simulation)
109. Benn DK. Frequent, low-dose, improved-contrast radiographic images with the use of narrow x-ray beams. *Oral Surg Oral Med Oral Pathol* 1992; **74:** 221–229.
(use of narrow x-ray beams to reduce irradiation for improved bone mass determination)
110. Bragger U, Burgin W, Lang NP, Buser D. Digital subtraction radiography for the assessment of changes in peri-implant bone density. *Int J Oral Maxillofac Implants* 1991; **6:** 160–166.
(three-stage a posteriori registration before digitization using chessboard, flicker and real-time subtraction modes)
111. Brocklebank LM, Kitsiou N, Richardson B, Still D. Development of equipment for standardization of DIGORA periapicals. *Dentomaxillofac Radiol* 1998; **27** (Suppl 1): 31 (abstr.).
(device for a priori registration of radiographs)
112. Burdea GC, Dunn SM, Immendorf CH, Mallik M. Real-time sensing of tooth position for dental digital subtraction radiography. *IEEE Trans Biomed Eng* 1991; **38:** 366–378.
(device for a priori registration using sensor measurement and robot arm)
113. Chalermwat P, El-Ghazawi T, Le Moigne J. Image registration by parts. *Proceedings Image Registration Workshop, Goddard Space Flight Center, Greenbelt, MD* 1997; 299–306.
(image registration using local rigid transforms)
114. Cohen ME, Roddy WC. A comparison of three statistics for detecting differences in digitized dental radiographs: A simulation study. *Dentomaxillofac Radiol* 1995; **24:** 179–184.
(quantitative measurements of subtraction images)
115. Davis M, Allen KM, Hausmann E. Effects of small angle discrepancies on interpretations of subtraction images. *Oral Surg Oral Med Oral Pathol* 1994; **78:** 397–400.
(angular differences greater than 1° significantly impair diagnostic accuracy)
116. Dunn SM, van der Stelt PF. Recognizing invariant geometric structure in dental radiographs. *Dentomaxillofac Radiol* 1992; **21:** 142–147.
(experimental proof that projective geometry is applicable to intra-oral radiographs)
117. Eickholz P, Dörfer C, Staehle HJ. Reproduzierbarkeit standartisierter Bißflügelaufnahmen bei Patienten mit fortgeschrittener Parodontitis. *Deutsche Zahnärztliche Zeitung* 1994; **49:** 398–402.
(precision of repositioning of individual bite blocks after 3 month is about 1.5°)
118. Fritsch DS, Pizer SM, Chaney EL, Liu A, Raghavan S, Shah T. Cores for image registration. *Proc SPIE* 1994; **2167:** 128–142.
(image registration on scene level of abstraction)
119. Giachetti A. Matching technique to compute image motion. *Image Vision Comput* 2000; **18:** 247–260.
(determination of motion vector fields in ultrasound images using local transforms)
120. Golub GH, van Loan CF. *Matrix computations*. Baltimore: The Johns Hopkins University Press, 1989.
(definition and solution of least-squares optimization by matrix computation)
121. Gröndahl HG, Gröndahl K, Okano T, Webber RL. Statistical contrast enhancement of subtraction images for radiographic caries diagnosis. *Oral Surg Oral Med Oral Pathol* 1982; **53:** 219–223.
(post-processing of subtraction images; effect of quantum noise)
122. Gröndahl HG, Gröndahl K, Webber RL. A digital subtraction technique for dental radiography. *Oral Surg Oral Med Oral Pathol* 1983; **55:** 96–102.
(one-stage a posteriori registration before digitization using real-time subtraction)
123. Hamadeh A, Cinquin P, Szeliski R, Lavallee S. Anatomy-based multimodal medical image registration for computer-inte-

grated surgery. *Proc SPIE* 1994; **2355:** 178 – 188.
(hybrid multi-modal image registration technique from 3D to 2D)

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(device for a priori registration using force-sensitive sensors)
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(for subtraction radiography, geometric misalignments should be limited to 0.7°)
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(quantitative measurements in subtraction images: digital image ratio)
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(device for a priori registration using cephalostat and ear plugs)
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(comparison of registration by varying starting positions on same images)
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(Email communication of technical details to determine the key code for[63]*)*
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(image processing on symbolic object level for identification of dental implants)
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(methodic paper on inter-comparison of registration algorithms)
132. Mol A, van der Stelt PF. Locating the periapical region in dental radiographs using digital image analysis. *Oral Surg Oral Med Oral Pathol* 1993; **75:** 373 – 382.
(definition of level of abstraction of features with iconic to symbolic image description: pixel, edges, boundaries, regions, objects, entity. In our terminology, we add the levels of raw data, texture, and scene and, furthermore, merge edges and boundaries to contrast level.)
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134. Phillips PJ, Huang J, Dunn SM. An efficient micrograph registration algorithm via sieve processes. *J Comput Ass Microscopy* 1996; **8:** 21 – 29.
(image registration on symbolic texture level applied to microscopy)
135. Rudolph DJ, White SC, Mankovich NJ. Influence of geometric distortion and exposure parameters on sensitivity of digital subtraction radiography. *Oral Surg Oral Med Oral Pathol* 1987; **64:** 631 – 637.
(for subtraction radiology, image degradation is evident after 1° of angular distortion but 3° misalignment may occur under clinical circumstances)
136. Rudolf DJ, White SC. Film-holding instruments for intraoral subtraction radiography. *Oral Med Oral Surg Oral Pathol* 1988; **65:** 767 – 772.
(precision of repositioning of individual bite blocks after 6 month is about 2.5°)
137. Shrout MK, Hildebolt CF, Vannier MW. Alignment errors in bitewing radiographs using uncoupled positioning devices. *Dentomaxillofac Radiol* 1993; **22:** 33 – 37.
(uncoupled positioning devices can be used when x-ray beam to film alignment error below 2.5° is acceptable)
138. Shrout MK, Weaver J, Potter BJ. Spatial resolution and angular alignment tolerance in radiometric analysis of alveolar bone change. *J Periodontol* 1996; **67:** 41 – 45.
(50 μm is apparently superior to 200 μm spatial resolution for digitization of dental films, and alignment variations up to 5° may be acceptable in clinical studies)
139. van der Stelt PF. Inference systems for automated image analysis. *Dentomaxillofac Radiol* 1992; **21:** 180 – 183.
(definition of levels of dimension of features with increasing complexity: pixel, line, region, texture, and time are labeled to be 1D, 2D, 3D, 4D, and 5D, respectively. They are followed by patient and condition without specific dimensions)
140. Versteeg CH, Sanderink GC, Geraets WG, van der Stelt PF. Impact of scale standardization on images of digital radiography systems. *Dentomaxillofac Radiol* 1997; **26:** 337 – 343.
(discussion of interpolation techniques applied to geometric image registration)
141. Vujovic N, Brzakovic D. Establishing the correspondence between control points in pairs of mammographic images. *IEEE Trans Image Proc* 1997; **6:** 1388 – 1399.
(image registration on symbolic texture level applied to mammography)
142. Wenzel A, Hintze H. Editorial review: The choice of gold standard for evaluating tests for caries diagnosis. *Dentomaxillofac Radiol* 1999; **28:** 132 – 136. (discussion **28:** 182 – 185 and **29:** 61 – 63).
(definition of three basic criteria any robust gold standard must fulfil)
143. Zappa U, Simona C, Graf H, van Aken J. In vivo determination of radiographic projection errors produced by a novel filmholder and an x-ray beam manipulator. *J Periodontol* 1991; **62:** 674 – 683.
(device for a priori registration)

Owing to the nature of this review, the order of citation of the references does not follow the usual style of *Dentomaxillofacial Radiology*. Because the references have been arranged in four sections, they have been cited in alphabetical order within each section. We believe readers will find this more helpful. A searchable database for these and other references created for this review is available in German Microsoft Access format from the first author. The database contains more than 320 entries; most include the abstracts.

Glossary

Technical terms and phrases often are used ambiguously. This glossary presents the most significant terms used in this review with a brief definition. When multiple definitions are possible, each term is defined in the context of subtraction imaging. A non-mathematical introduction to digital image processing combined with a more detailed glossary of technical terms is given by Baxes[107] or by the free online dictionary of computing at the Imperial College, London, which also links to other web-dictionaries: URL: http://wombat.doc.ic.ac.uk/foldoc.

1st generation subtraction: A system for photographic subtraction of standardized radiographic films.
2nd generation subtraction: A system for digital subtraction of standardized radiographs, where geometric alignment is done manually before digitization.
3rd generation subtraction: A system for digital subtraction of radiographs where geometric and contrast registration are performed by computer after digitization.
Affine transform: A linear geometric transformation that, in two dimensions, is defined by six parameters. Affine transforms allow rotation, translation, scaling as well as regular reflection and shearing. The pixel coordinates of the output image (x',y') are computed from those of the input image (x,y) by

$$\begin{matrix} x' = a_1 + a_2x + a_3y \\ y' = a_4 + a_5x + a_6y \end{matrix}, a_i \in \mathbb{R} \tag{6}$$

To avoid misunderstanding, other terms for affine transforms, such as planar projective or weak perspective, should not be used.
Algorithm: A finite series of clearly defined logical steps to solving a problem: in particular, a mathematically specific technique used to implement a certain image processing or analysis operation (named after an Iranian mathematician, Al-Khawarizmi).
Aperture: The light-gathering opening in an imaging system, e.g. a lens, but also the measure of the light-gathering opening in a lens.
***A posteriori* registration:** A method that is able to register images without including special devices or protocols in the imaging process. The registration is done after image acquisition (see Section 2.2).
***A priori* registration:** A method assuming that at the time of image recording it is known that the image will be used for subtraction. Special devices or protocols are included in the imaging process in order to enable registration which is done before image acquisition (see Section 2.1).
Aspect ratio: The ratio of the horizontal to vertical dimension of an image or pixel generally stated as $x:y$.
Baseline acquisition: The point of time when the reference radiograph in a longitudinal study is obtained.
Bending: *see* film bending.
Bilinear transform: A geometric transform that, in two dimensions, is defined by eight parameters. Affine transforms (Equation 6) are completely contained within by bilinear transforms whereas projective transforms (Equation 9) are not. The pixel coordinates of the output image (x',y') are computed from those of the input image (x,y) by

$$\begin{matrix} x' = a_1 + a_2x + a_3y + a_4xy \\ y' = a_5 + a_6x + a_7y + a_8xy \end{matrix}, a_i \in \mathbb{R} \tag{7}$$

Binary image: An image composed of only black and white brightnesses.
Biquadratic transform: A quadratic geometric transform that, in two dimensions, is defined by 18 parameters. Bilinear transforms (Equation 7) are completely contained within biquadratic transforms whereas projective transforms (Equation 9) are not. The pixel coordinates of the output image (x',y') are computed from those of the input image (x,y) by

$$\begin{matrix} x' = a_1 + a_2x + a_3y + a_4xy + a_5x^2 + a_6y^2 + a_7xy^2 + a_8x^2y + a_9x^2y^2 \\ y' = a_{10} + a_{11}x + a_{12}y + a_{13}xy + a_{14}x^2 + a_{15} + a_{16}xy^2 + a_{17}x^2y + a_{18}x^2y^2 \end{matrix}, a_i \in \mathbb{R} \tag{8}$$

Brightness: The quantity of light assigned to a pixel in a digital image. In comparison, *intensity* refers to the quantity of light or X-rays actually reflected or transmitted.
Brightness histogram: *see* histogram.
Brute force: A primitive programming style in which the programmer relies on the computer's processing power instead of using his own intelligence to simplify the problem.

Calibration: *see* contrast calibration *or* geometric calibration.

Category: The terminology used in this paper is that categories (origin, level of abstraction, and linkage of features) subdivide a criterion (feature).

Computer-based registration: An *a posteriori* registration method where registration is performed by a computer after image acquisition and digitization (see Section 2.3).

Contrast: The differences of brightness within a digital image, e.g. the standard deviation of the gray-values of the pixels. Global contrast refers to the entire image whereas local contrast addresses only a small neighborhood of pixels.

Contrast calibration: Computational compensation for contrast distortions caused by image acquisition, e.g. gamma correction, not to be confused with *contrast registration*.

Contrast correction: *see* contrast registration.

Contrast distortion: A non-linear modification of intensities caused by the imaging process. For example, gamma correction is needed to compensate for the non-linear brightness display characteristics of a cathode-ray tube or to calibrate photometric distortions of X-ray films.

Contrast registration: A registration technique that operates on the value range of an image. It is characterized by the categories, origin of features, model of transform, and interaction of procedure (see Sections 2.3.2 and 3.2).

Control points: Multiple points placed on two images to control the geometric registration process. The points within baseline and follow-up radiographs represent before and after locations of the transform, respectively.

Convolution: A mathematical function of weighted average computing, e.g. for contrast or edge enhancement. Usually an image is convolved with a small-sized convolution mask.

Convolution mask: A group of pixels covering the kernel of the input image in a convolution process. The convolution mask is not to be confused with a *masking image*.

Correlation: A mathematical comparison function used to assess the similarity of 1D functions or 2D images. For symmetrical images or functions, correlation equals convolution.

Criterion: The terminology used in this paper is that a criterion (dimension, feature, transform, interaction, or contrast) is divided into a number of categories.

DFT: *see* discrete Fourier transformation.

Differencing: *see* subtraction.

Difference image: *see* subtraction image.

Digital image: An image composed of discrete pixels, each having an associated discrete brightness value.

Digital image analysis: The technique of processing digital images on higher levels of abstraction, i.e. texture, region, object, or scene descriptions. In comparison, *digital image processing* directly addresses the discrete digital brightness quantities.

Digital image processing: The technique of processing images while they are in the form of discrete digital brightness quantities. In comparison, *digital image analysis* is the technique of processing images on higher levels of abstraction.

Digitization: Sampling and quantizing an analog signal to create a digital image.

Discrete Fourier transformation: A Fourier transform, specialized to the case where the abscissas are integers. A common implementation of the DFT is the FFT.

Distortion: *see* geometric distortion *or* contrast distortion.

Elastic transform: A geometric transform of high complexity that is approximated by a polynomial equation of high or infinite order.

Fast Fourier transformation: An algorithmically faster version of the DFT. The image dimensions must be integer multiples of two.

Feature: A particular property.

FFT: *see* fast Fourier transform.

Film bending: Deformation of the film that results in an elastic transform between baseline and follow-up radiographs.

Film tilting: Rigid misalignment of serial radiographs.

Follow-up acquisition: The point of time when a subsequent radiograph in a longitudinal study is obtained.

Fourier domain: *see* frequency domain.

Fourier transformation: A frequency transform that decomposes a spatial image into a set of sinusoidal frequency component functions. High local image contrast is related to high frequency components.

Frame grabber: A computer peripheral providing the acquisition, storage, and display of digital images based on an analogue video signal.

Free-hand subtraction: 3rd generation subtraction performed without individual adjustment aids. However, stents or other non-individual devices may be used to control geometry.

Frequency domain: The Fourier transform converts an image from its spatial into its complex frequency domain (Fourier domain) where pixel brightness correspond to the spatial frequency content of the image and pixel phases to their relative locations.

Gamma correction: A non-linear correction of brightnesses. The parameter gamma determines the degree of freedom of the power function.

Geometric calibration: Computational compensation of geometric distortions caused by image acquisition, such as barrel distortion, not to be confused with *geometric registration*.

Geometric distortion: A non-linear modification of geometrical distances caused by the imaging process. For example, barrel distortion is often induced by lenses and causes an acquired image to appear to pucker towards the center.

Geometric registration: A registration technique that addresses the definition range of an image. It is characterized by the criteria: dimension, feature, transform and interaction (see Sections 2.3.1 and 3.1).

Geometric transform: An operation that transforms the spatial characteristics of an image. Its elasticity increases hierarchically from shift, rigid, RST, and affine to projective or bilinear. In order to define these unambiguously, geometric transforms must be determined by their smallest possible degree of freedom.

Grayscale: The number of gray-levels that represent the brightness in a digital image.

Grayscale image: An image composed of gray-level brightness.

Gray value: A numerical representation of a certain gray-level brightness.

Histogram: A graphical representation of the number of pixels in an image at each gray-level. In other words, a histogram visualizes the *a priori* probabilities (statistics) of the occurrence of each gray value.

Image analysis: *see* digital image analysis.

Image combination: Any operation that associates two or more images pixel by pixel such as subtraction. A masking image may be used for local adaptive control of the combination.

Imaging: The process of image capture.

Imaging geometry: The geometric relation of the imaging source or viewpoint, the object to be captured, and the view or imaging plane. The imaging geometry establishes the spatial resolution requirements of the imaging sensor.

Imaging plane: The plane perpendicular to the line between the observer's viewpoint and the displayed object. In an X-ray projection, the imaging plane is perpendicular to the central ray.

Input image: An image that is processed in a digital image processing or analysis system.

Intensity: The quantity of light that is actually reflected or transmitted from a physical setting. In comparison, *brightness* refers to the quantity of light assigned to a pixel in a digital image.

Interpolation: A process that transforms a discrete into a continuous image where the brightness at any location between the discrete grid is calculated from its discrete neighbors on the grid by weighted averaging. For geometric transforms of digital images, the data is resampled after its interpolation.

Invariant feature: The ability of a feature measure to stay constant even when an identical object appears differently such as rotated or scaled. Invariant image features are utilized for direct determination of registration parameters.

Irradiation geometry: *see* imaging geometry.

Kernel: The group of input image pixels at the actual mask position used in the spatial convolution process.

Landmark: A characteristic anatomical location that is often used to define control points for point mapping processes.

Look-up table: A hard- or software implementation of a brightness mapping function. For every possible input pixel brightness, a corresponding output pixel brightness is stored in the LUT.

LUT: *see* look-up table.

Least-squares: A technique of error distribution that minimizes the mean squared error.

Mapping: A function that converts the input pixel location or brightness into the output pixel location or brightness, respectively.

Matching: *see* geometry registration.

Masking image: Usually a binary image of equal size to the input image that is used to code for each pixel position whether this pixel is concerned or not in the desired operation.

Motion correction: *see* geometry registration.

MSE: Abbreviation for mean squared error.

Mutual information: A quantity of information-theory not only assessing the agreement of a random process with another but also its disagreement from all other processes. Superior classification algorithms are based on mutual information.

Noise: *see* structured noise *or* random noise.

Optical resolution: The overall capability of an imaging system to resolve spatial details in an image scene, not to be confused with the *spatial resolution* and the *spatial density* of the sensor element.

Output image: An image that results from digital image processing or analysis.

Perspective transform: *see* projective transform.

Photometric distortion: A form of brightness distortion caused by incongruencies in the light response of an image receptor, yielding brightnesses that do not accurately represent the intensities of the original scene.
Picture element: *see* pixel.
Pixel: Artificial word created from picture and element to describe the smallest discrete spatial component of 2D digital images. For volume data sets, pixels also are called voxels.
Planar projective transform: *see* affine transform.
Point mapping: A geometric transform that is done by exact mapping of spatial pixel locations based on control points.
Point pattern matching: A geometric transform that is done by approximate mapping of spatial pixel locations. The elasticity of transform is determined before its parameters are computed to match the point patterns.
Power spectrum: A real image that assigns the squared amplitude of the complex Fourier transform to pixel brightness.
Projective transform: A 2D geometric transform defined by eight parameters. Projective transforms occur in X-ray imaging modalities such as intra-oral radiography and cause objects in an image to appear trapezoidal rather than square due to foreshortening. The pixel coordinates of the output image (x',y') are computed from those of the input image (x,y) by

$$x' = \frac{a_1 + a_2 x + a_3 y}{1 + a_7 x + a_8 y},\ y' = \frac{a_4 + a_5 x + a_6 y}{1 + a_7 x + a_8 y}, a_i \in \mathbb{R} \tag{9}$$

Prospective registration: *see a priori* registration.
Quantization: The process of converting discrete image samples to digital quantities of brightness following the sampling process.
Random noise: An additive part of noise in an image resulting from the imaging process.
Range data: Assuming parallel beams, the brightness of a pixel is used to code the distance between the receptor and the surface of an object.
Real-time processing: The ability to carry out digital image processing or analysis on the entire image as quickly as new images are available. Based on video processing, real-time is typically 0.04 s while for single images, real-time means without notable delay.
Reference image: An image acquired at baseline used as standard in longitudinal studies.
Reflection: A geometric operation that mirrors an image from left to right or upside down.
Registration: The process of determining a relationship between the content of two images including the projection of one image onto the geometry of the other by interpolation.
Retrospective registration: *see a posteriori* registration.
Rigid transform: A linear geometric transform of a rigid body allows rotation and translation. In two dimensions, rigid transforms are described by three parameters, where α and (γ_1,γ_2) denotes the rotation angle and translation vector, respectively. The pixel coordinates of the output image (x',y') are computed from those of the input image (x,y) by

$$\begin{array}{l} x' = \cos(\alpha)\cdot x - \sin(\alpha)\cdot y + \gamma_1 \\ y' = \sin(\alpha)\cdot x + \cos(\alpha)\cdot y + \gamma_2 \end{array}, \alpha, \gamma_i \in \mathbb{R} \tag{10}$$

Rotation: A geometric operation that rotates an image about a predetermined point through a desired angle. Rotations are comprised within rigid transforms and all other geometric transforms of higher degree of freedom (elasticity).
Rubber sheet transform: *see* warping.
RST: Abbreviation for rotation, scaling, and translation.
RST transform: A linear geometric transform comprising rotation, scaling, and translation. In two dimensions, RST transforms are described by four parameters, where β denotes the scale. The pixel coordinates of the output image (x',y') are computed from those of the input image (x,y) by

$$\begin{array}{l} x' = \beta\cdot\cos(\alpha)\cdot x - \beta\cdot\sin(\alpha)\cdot y + \gamma_1 \\ y' = \beta\cdot\sin(\alpha)\cdot x + \beta\cdot\cos(\alpha)\cdot y + \gamma_2 \end{array}, \alpha, \beta, \gamma_i \in \mathbb{R} \tag{11}$$

Sampling: The process of dividing an analog signal into discrete representatives (samples) preceding the quantization process.
Scaling: A geometric operation that changes the size of objects in an image. Scales are comprised within an RST transform and other geometric transforms of higher degrees of freedom.

Segment: An agglomeration of neighboring pixels.

Segmentation: The process of subdividing an image into segments.

Shearing: A geometric operation that shifts image lines or rows corresponding to their line or row number. Shearing is contained within affine transforms and all other geometric transforms of higher elasticity.

Shift transform: A linear geometric transform that is restricted to translations. In two dimensions, shifts are determined by two parameters. The shift transform is contained within all the other transformation models discussed in this review. The pixel coordinates of the output image (x',y') are computed from those of the input image (x,y) by

$$\begin{matrix} x' = x + \gamma_1 \\ y' = y + \gamma_2 \end{matrix}, \gamma_i \in \mathbb{R} \tag{12}$$

Software application: A computer program used to interact with and implement algorithms for digital image processing and analysis.

Spatial: The relation to the 2D nature of an image.

Spatial convolution: *see* convolution.

Spatial density: The number of pixels in a sensor related to its dimensions, not to be confused with the *spatial resolution* of the imaging system.

Spatial distortion: *see* geometric distortion.

Spatial domain: The natural form of an image where pixel brightness corresponds directly to spatial image brightness.

Spatial resolution: The smaller of the measurements of spatial density of the sensor and optical resolution of the imaging system. However, the spatial resolution is typically assumed to be equal to the spatial density of the sensor.

Spatial frequency: The rate at which a spatial detail transits from dark to light or vice versa. A fine detail has high spatial frequency content (high contrast) while a coarse detail has low spatial frequency (low contrast).

Structured noise: In serial radiographs, that part of image information describing unaltered morphological structures and covering temporal changes. Hence, subtraction is applied to reduce structured noise.

Subtraction: An operation that subtracts one image from another, pixel by pixel. Typically, each image is of the same scene but acquired at different times, e.g. at baseline and follow-up.

Subtraction image: The image resulting from a pixel-by-pixel subtraction of one image from another. Usually, pixels without any brightness change are assigned to medium gray in a subtraction image.

TACT: *see* tuned aperture computed tomography.

Texture: A measure of the pseudo-periodic variation of local pixel brightness quantifying properties such as smoothness, coarseness, and regularity.

Tilting: *see* film tilting.

Tomosynthesis: The alignment of 2D projections acquired by moving the receptor in the opposite direction to the X-ray tube, resulting in crosssections of a 3D object.

Translation: A geometric operation that shifts an image left, right, up, or down.

Tuned aperture computed tomography: A tomographic X-ray modality that is based on an arbitrary number of projections, each of which is acquired with non-standardized geometry.

Volume element: *see* voxel.

Voxel: An artificial word created from volume and element to describe the smallest discrete spatial component of 3D digital images.

Warping: A geometric operation of high elasticity that contorts an image, often performed with the aid of control points.

Weak perspective transform: *see* affine transform.